S.S.L.C

Yakub Koyyur
GHS nada
Belthangady taluk
D.K. – 574214
Email:
yhokkila@gmail.com

MATHS

English Medium

Part-2

All Solutions

Based on new syllubus

Available in:ykoyyur.blogspot.com

Content

Part –2

9 Polynomials

Degree of the polynomial:

p(x) is a polynomial in x, the highest power of x in p(x) is called the degree of the polynomial p(x).

Examples:

4x + 2 is a polynomial in the variable x ofdegree 1.

A polynomial of degree 1 is called a **linear polynomial.**

$2y^2 - 3y + 4$ is a polynomial in the variable y of degree

A polynomial of degree 2 is called a **quadratic polynomial.**

quadratic polynomial in x is of the form $\mathbf{ax^2 + bx + c}$, where a, b, c are real numbers $a \neq 0$.

is a polynomial in the variable x of degree 3

$5x^3 - 4x^2 + 2 - \sqrt{2}$ is a polynomial in the variable x of degree 3

A polynomial of degree 3 is called a **cubic polynomial**. General form of a cubic polynomial is

$\mathbf{ax^3 + bx^2 + cx + d}$

Where a, b, c, d are real numbers and $a \neq 0$

[$7u^6 - \frac{3}{2}u^4 + 4u^2 + u - 8$ is a polynomial of variable x and the degree of this polynomial is 6]

Example:: $\sqrt{x} + 1$, $\frac{2}{x}$, $\frac{1}{x^3+x^2-1}$

If p(x) is a polynomial in x, and if k is any real number, then the value obtained by replacing x by k in p(x), is called the value of p(x) at x = k, and is denoted by p(k).

What is the value of $p(x) = x^2 - 3x - 4$ when x = - 1 ?

$p(-1) = (-1)^2 - 3(-1) - 4 = 0$

Similarlly, p(4) = (4)2 - 3(4) - 4 = 0

As p(-1) = 0 and p(4) = 0 –1 and 4 are called the zeros of the polynomial $x^2 - 3x - 4$

If k is a real number such that p(k) = 0 then k is called the Zeros of the polynomial p(x)

If k is the zero of the polynomial p(x) = ax + b then $p(k) = ak + b = 0 \Rightarrow k = -\frac{b}{a}$

The zero of the lenear equation ax + b is $-\frac{b}{a}$

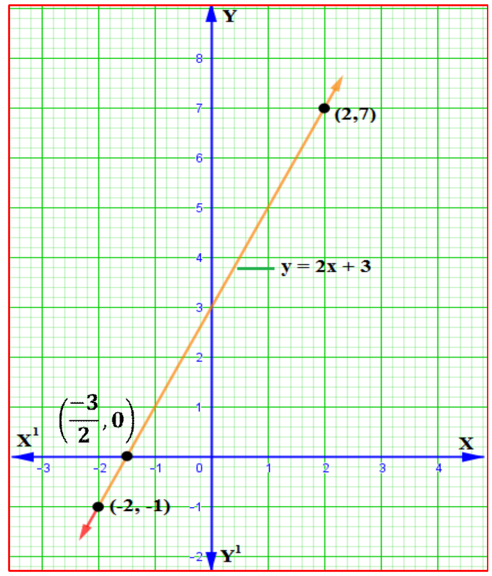
9.2 Geometrical Meaning of the Zeroes of a Polynomial

(i) Linear Polynomial

Example y = 2x + 3

x	**-2**	**2**
y	-1	7

The graph of $y = 2x + 3$ is a straight line passing through the points $(-2, -1)$ and $(2, 7)$.
The graph of $y = 2x + 3$ intersects X – axis at the points $\left(\frac{-3}{2},\ 0\right)$,Thus the zero of the polynomial 2x+3 is the x-coordinate of the point where the graph of $y = 2x + 3$ intersects the axis
$\therefore\ \frac{-3}{2}$ is the zero of the linear polynomial $y = 2x + 3$
$\therefore$The linear polynomial $ax + b$ $(a \neq 0)$ has exactly one zero,namely the x-coordinate of the point where the graph of $y = ax + b$ intersects the axis

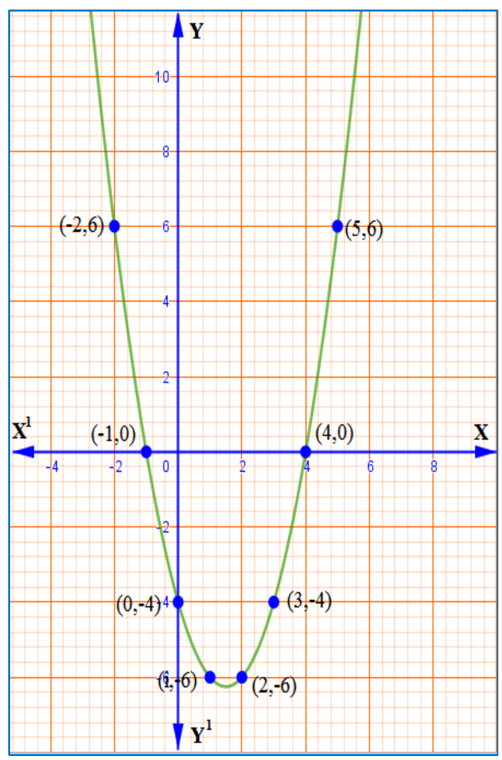


(i) Quadratic Polynomials:

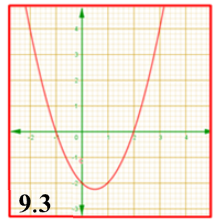
Example: $y = x^2 - 3x - 4$

x	-2	-1	0	1	2	3	4	5
y	6	0	-4	-6	-6	-4	0	6

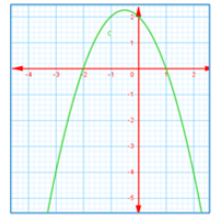
If we locate the points listed above on a graph paper and draw the graph, it will actually look like the one given in Fig. In fact, for any quadratic polynomial $ax^2 + bx + c$, a 0, the graph of the corresponding equation $y = ax^2 + bx + c$ has one of the two shapes either open upwards or open downwards depending on whether $a > 0$ or $a < 0$. (Thesecurves are called parabolas.) –1 and 4 are the x-coordinates of the points where the graph of $y = x^2 - 3x - 4$ intersects the x-axis. Thus, the zeroes of the quadratic polynomial $x^2 - 3x - 4$ are x-coordinates of the points where the graph of $y = x^2 - 3x - 4$ intersects the x-axis. This fact is true for any quadratic polynomial, i.e., the zeroes of a quadratic polynomial $ax^2 + bx + c$, a 0, are precisely the x-coordinates of the points where the parabola representing $y = ax^2 + bx + c$ intersects the x-axis



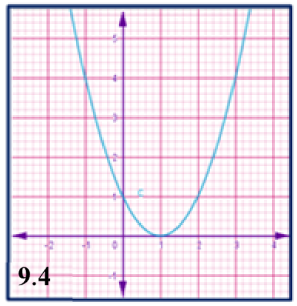
Case (i) : Here, the graph cuts x-axis at two distinct points A and A^1. The x-coordinates of A and A^1 are the two zeroes of the quadratic polynomial $x^2 + bx + c$ in this case (see Fig. 9.3).



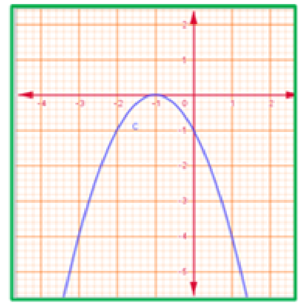
9.3



Case (ii) : Here, the graph cuts the x-axis at exactly one point, i.e., at two coincident points. So, the two points A and A^1 of Case (i) coincide here to become one point A (see Fig. 9.4).
The x –coordinate of A is the only zero for the quadratic polynomial $ax^2 + bx + c$ in this case.

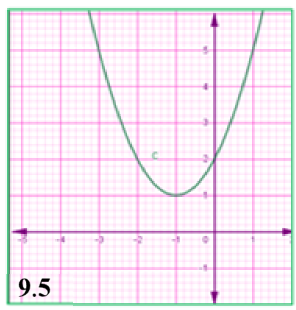


9.4



Case (iii) : Here, the graph is either completely above the x-axis or completely below the x-axis. So, it does not cut the x-axis at any point (see Fig. 9.5).

So, the quadratic polynomial $ax^2 + bx + c$ has no zero in this case.



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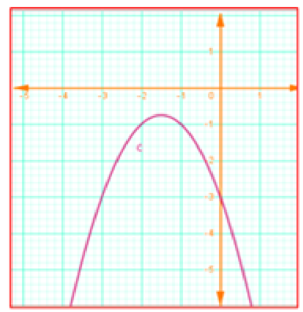


So, you can see geometrically that a quadratic polynomial can have either two distinct zeroes or two equal zeroes (i.e., one zero), or no zero. This also means that a polynomial of degree 2 has atmost two zeroes.

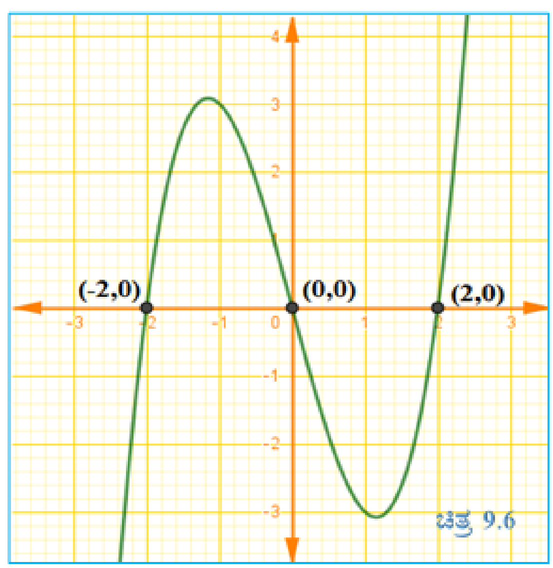
Cubic Polynomials:

Example: $y = x^3 - 4x$

x	-2	-1	0	1	2
y	0	3	0	-3	0

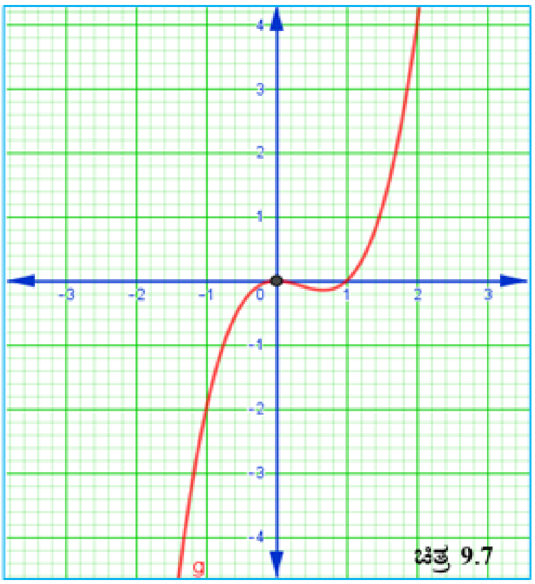
Locating the points of the table on a graph paper and drawing the graph, we see that the graph of $y = x^3 – 4x$ actually looks like the one given in fig 9.6

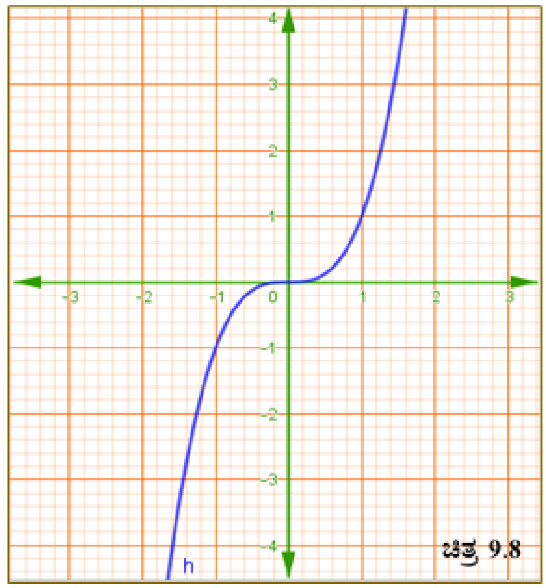
We see from the table above that – 2, 0 and 2 re zeroes of the cubic polynomial $x^3 – 4x$. Observe that –2, 0 and 2 are, in fact, the x-coordinates of the only points where the graph of $y = x^3 – 4x$ intersects the x-axis. Since the curve meets the x-axis in only these 3 points, their x-coordinates are the only zeroes of the polynomial



ಚಿತ್ರ 9.6

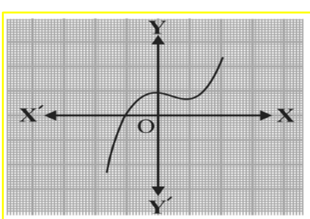
Let us take a few more examples. Consider the cubic polynomials x^3 and $x^3 - x^2$. We draw the graphs of $y = x^3$ and $y = x^3 - x^2$ in Fig. 9.7 and Fig. 9.8 respectively. Note that 0 is the only zero of the polynomial x^3. Also, from Fig. 9.7, you can see that 0 is the x-coordinate of the only point where the graph of $y = x^3$ intersects the x-axis. Similarly, since $x^3 - x^2 = x^2 (x - 1)$, 0 and 1 are the only zeroes of the polynomial $x^3 - x^2$. Also, from Fig. 9.8, these values are the x-coordinates of the only points where the graph of $y = x^3 - x^2$ intersects the x-axis



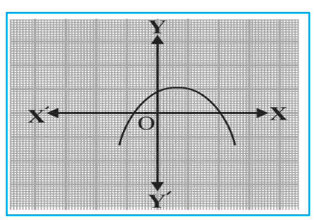


From the examples above, we see that there are at most 3 zeroes for any cubic polynomial. In other words, any polynomial of degree 3 can have at most three zeroes

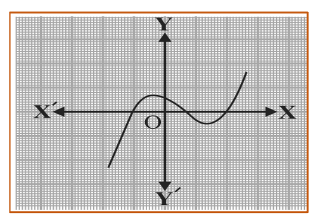
Example 1 : Look at the graphs in Fig. 9.9 given below. Each is the graph of y = p(x), where p(x) is a polynomial. For each of the graphs, find the number of zeroes of p(x).



(i)



(ii)



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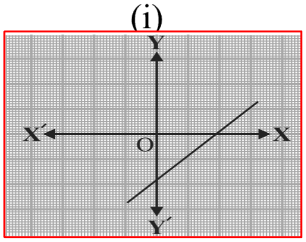

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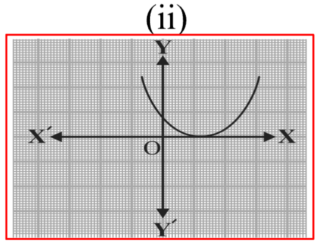

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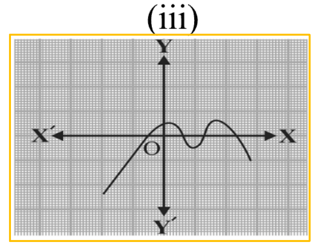

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Solution :

(i) The number of zeroes is 1 as the graph intersects the x-axis at one point only.
(ii) The number of zeroes is 2 as the graph intersects the x-axis at two points.
(iii) The number of zeroes is 3 as the graph intersects the x-axis at three points
(iv) The number of zeroes is 1 as the graph intersects the x-axis at one point only.
(v) The number of zeroes is 1. as the graph intersects the x-axis at one point only.
(vi) The number of zeroes is 4. as the graph intersects the x-axis at four points

Exercise 9.1

1. The graphs of y = p(x) are given in Fig. 9.10 below, for some polynomials p(x). Find the number of zeroes of p(x), in each case.

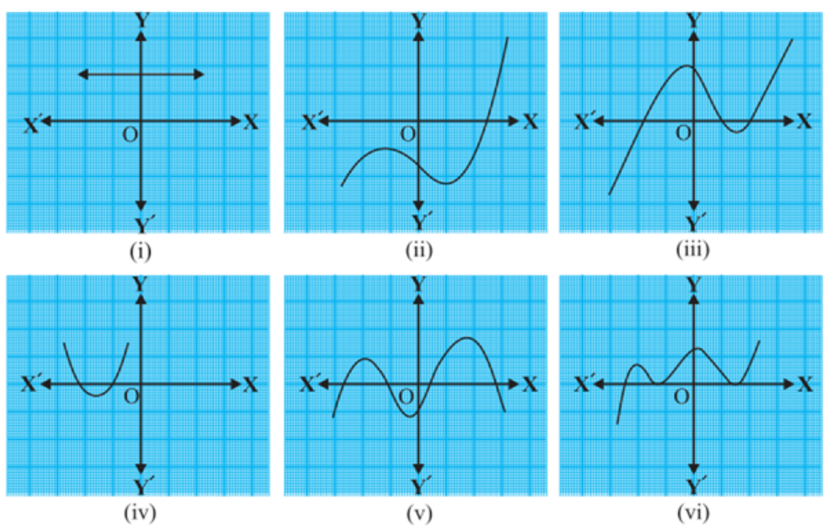


Fig. 9.10

(i) The number of zeroes is 0 as the graph not intersects the x-axis
(ii) The number of zeroes is 1 as the graph intersects the x-axis at one point only.
(iii)The number of zeroes is 3 as the graph intersects the x-axis at three points .
(iv)The number of zeroes is 2 as the graph intersects the x-axis at two points.
(v) The number of zeroes is 4 as the graph intersects the x-axis at four points.
(vi) The number of zeroes is 3 as the graph intersects the x-axis at three points.

9.3 Relationship between Zeroes and Coefficients of a Polynomial

α and β are the zeros of the polynomial $p(x) = ax^2 + bx + c, \quad a \neq 0$
$(x - \alpha)$ and $(x - \beta)$ are the factors of $p(x)$.

Sum of Zeros $\alpha + \beta = \frac{-b}{a}$ Product of Zeros $\alpha\beta = \frac{c}{a}$

Example 2 : Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$, and verify the relationship between the zeroes and the coefficients

Solution: $x^2 + 7x + 10 = x^2 + 5x + 2x + 10$

$= x(x+5) + 2(x+5) = (x+2)(x+5)$

$\therefore$ The value of $x^2 + 7x + 10$ is zero when $x = -2$ or $x = -5$

$\therefore -2$ and -5 are the zeros of $x^2 + 7x + 10$

Sum of the zeros $= (-2) + (-5) = -7 = \frac{-7}{1} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeros $= (-2) \times (-5) = 10 = \frac{10}{1} = \frac{\text{Constant}}{\text{Coefficient of } x^2}$

Example 3 : Find the zeroes of the polynomial $x^2 - 3$ and verify the relationship between the zeroes and the coefficients

Solution: $a^2 - b^2 = (a - b)(a + b)$

$\therefore x^2 - 3 = (x - \sqrt{3})(x + \sqrt{3})$

$\therefore \sqrt{3}$ and $-\sqrt{3}$ are the zeros of $x^2 - 3$

Sum of the zeros $= \sqrt{3} + -\sqrt{3} = 0 = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeros $= (\sqrt{3})(-\sqrt{3}) = -3 = \frac{-3}{1} = \frac{\text{Constant}}{\text{Coefficient of } x^2}$

Example 4 : Find a quadratic polynomial, the sum and product of whose zeroes are –3 and 2, respectively.

Solution: Let the required polynomial be $ax^2 + bx + c$ and its zeros are α and β.

$\therefore \alpha + \beta = -3 = \frac{-b}{a}$ and $\alpha\beta = 2 = \frac{c}{a}$

$\Rightarrow$ If $a = 1$ then $b = 3$ and $c = 2$

$\therefore$ Quadratic polynomial $= x^2 + 3x + 2$

The relation between the zeros and the coefficients of Cubic polynomials:

If α, β, γ are the zeros of the cubic polynomia $ax^3 + bx^2 + cx + d$ then

$$\alpha + \beta + \gamma = \frac{-b}{a}; \quad \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}; \quad \alpha\beta\gamma = \frac{-d}{a}$$

Exercise 9.2

1. **Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.**

(i) $x^2 - 2x - 8$ **(ii)** $4s^2 - 4s - 1$ **(iii)** $6x^2 - 3 - 7x$

(iv) $4u^2 - 8u$ **(v)** $t^2 - 15$ **(vi)** $3x^2 - x - 4$

(i) $x^2 - 2x - 8 = x^2 - 4x + 2x - 8 = (x-4) + 2(x-4) = (x-4)(x+2)$

$\Rightarrow x = 4$ and $x = -2$ are the zeros of polynomial $x^2 - 2x - 8$

Sum of the zeros $= 4 + (-2) = 2 = \frac{-(-2)}{1} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeros $= (4)(-2) = -8 = \frac{-8}{1} = \frac{\text{Constant}}{\text{Coefficient of } x^2}$

(ii) $4s^2 - 4s + 1 = 4s^2 - 2s - 2s + 1 = 2s(s-1) - 1(2s-1) = (2s-1)(2s-1)$

$\Rightarrow s = \frac{1}{2}$ and $s = \frac{1}{2}$ are the zeros of the polynomial $4s^2 - 4s + 1$

Sum of the zeros $= \frac{1}{2} + \frac{1}{2} = 1 = \frac{-(-4)}{4} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeros $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{\text{Constant}}{\text{Coefficient of } x^2}$

(iii) $6x^2 - 3 - 7x = 6x^2 - 7x - 3 = 6x^2 - 9x + 2x - 3 = 3x(2x-3) + 1(2x-3) = (3x+1)(2x-3)$

$\Rightarrow x = -\frac{1}{3}$ and $x = \frac{3}{2}$ are the zeros of the polynomial $6x^2 - 3 - 7x$

Sum of the zeros $= -\frac{1}{3} + \frac{3}{2} = 1 = \frac{-2+9}{6} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeros $= -\frac{1}{3} \times \frac{3}{2} = \frac{-3}{6} = \frac{\text{Constant}}{\text{Coefficient of } x^2}$

(iv) $4u^2 + 8u$ $= 4u^2 + 8u + 0 = 4u(u + 2)$

$\Rightarrow u = 0$ and $u = -2$ are the zeros of the polynomial $4u^2 + 8u$

Sum of the zeros $= 0 + (-2) = -2 = \frac{-(8)}{4} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeros $= 0 \times -2 = 0 = \frac{0}{4} = \frac{\text{Constant}}{\text{Coefficient of } x^2}$

(v) $t^2 - 15$ $= t^2 - 0.t - 15 = (t - \sqrt{15})(t + \sqrt{15})$

$\Rightarrow t = \sqrt{15}$ and $t = -\sqrt{15}$ are the zeros of the polynomial $t^2 - 15$

Sum of the zeros $= \sqrt{15} + (-\sqrt{15}) = 0 = \frac{0}{1} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeros $= \sqrt{15} \times (-\sqrt{15}) = -15 = \frac{-15}{1} = \frac{\text{Constant}}{\text{Coefficient of } x^2}$

(vi) $3x^2 - x - 4$ $= 3x^2 - 4x + 3x - 4 = x(3x-4)+1(3x-4) = (3x - 4)(x + 1)$

$\Rightarrow x = \frac{4}{3}$ and $x = -1$ are the zeros of the polynomial $3x^2 - x - 4$

Sum of the zeros $= \frac{4}{3} + (-1) = \frac{4-3}{3} = \frac{1}{3} = \frac{-(-1)}{3} \quad \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeros $= \frac{4}{3} + (-1) = -\frac{4}{3} = \frac{-4}{1} = \frac{\text{Constant}}{\text{Coefficient of } x^2}$

1. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively. (i) $\frac{1}{4}$, -1 (ii) $\sqrt{2}$, $\frac{1}{3}$ (iii) 0, $\sqrt{5}$ (iv) 1 , 1 (v) $-\frac{1}{4}$, $\frac{1}{4}$ (vi) 4 , 1

(i) $\frac{1}{4}$, -1 - Let the required polynomial be $ax^2 + bx + c$ and its zeros are α and β.

$\alpha + \beta = \frac{1}{4} = \frac{-(-1)}{4} = \frac{-b}{a}$ and $\alpha\beta = -1 = \frac{-4}{4} = \frac{c}{a}$

$\Rightarrow a = 4, b = -1$ and $c = -4$

$\therefore$ The required polynomial is $4x^2 - x - 4$

(ii) $\sqrt{2}$, $\frac{1}{3}$ - Let the required polynomial be $ax^2 + bx + c$ and its zeros are α and β.

$\alpha + \beta = \sqrt{2} = \frac{-(3\sqrt{2})}{3} = \frac{-b}{a}$ and $\alpha\beta = \frac{1}{3} = \frac{c}{a}$

$\Rightarrow a = 3, b = -3\sqrt{2}$ and $c = 1$

$\therefore$ The required polynomial is $3x^2 - 3\sqrt{2} + 1$

(iii) 0, $\sqrt{5}$ – Let the required polynomial be $ax^2 + bx + c$ and its zeros are α and β.

$\alpha + \beta = 0 = \frac{0}{1} = \frac{-b}{a}$ and $\alpha\beta = \sqrt{5} = \frac{\sqrt{5}}{1} = \frac{c}{a}$

$\Rightarrow a = 1, b = 0$ and $c = \sqrt{5}$

$\therefore$ The required polynomial is $x^2 + \sqrt{5}$

(iv) 1, 1 - Let the required polynomial be $ax^2 + bx + c$ and its zeros are α and β.

$\alpha + \beta = 1 = \frac{-(-1)}{1} = \frac{-b}{a}$ and $\alpha\beta = 1 = \frac{1}{1} = \frac{c}{a}$

$\Rightarrow a = 1, b = -1$ and $c = 1$

$\therefore$ The required polynomial is $x^2 - x + 1$

(v) $-\frac{1}{4}$, $\frac{1}{4}$ - Let the required polynomial be $ax^2 + bx + c$ and its zeros are α and β.

$\alpha + \beta = -\frac{1}{4} = \frac{-1}{4} = \frac{-b}{a}$ and $\alpha\beta = \frac{1}{4} = \frac{1}{4} = \frac{c}{a}$

$\Rightarrow$ a = 4, b = 1 and c = 1

$\therefore$ The required polynomial is $4x^2 + x + 1$

(vi) 4,1 - Let the required polynomial be $ax^2 + bx + c$ and its zeros are α and β

$\alpha + \beta = 4 = \frac{-(-4)}{1} = \frac{-b}{a}$ and $\alpha\beta = 1 = \frac{1}{1} = \frac{c}{a}$

$\Rightarrow$ a = 1, b = -4 and c = 1

$\therefore$ The required polynomial is $x^2 - 4x + 1$

9.4 Division Algorithm for Polynomials:

Let the zero of $x^3 - 3x^2 - x + 3$ is 1, then the factor is $(x - 1)$

Now, divide $x^3 - 3x^2 - x + 3$ by the factor $(x - 1)$ then the quotient is $x^2 - 2x - 3$.

By factorising $x^3 - 3x^2 - x + 3$ we get the factors $= (x - 1)(x + 1)(x - 3)$

$\therefore$ the zeros of the polynomial $x^3 - 3x^2 - x + 3$ is 1, –1 and 3

Example 6 : Divide $2x^2 + 3x + 1$ by $x + 2$.

$x + 2$	$2x^2 + 3x + 1$	$2x - 1$
	$2x^2 + 4x$	
	$-x + 1$ $-x - 2$	
	$+3$	

Solution: Note that we stop the division process when either the remainder is zero or its degree is less than the degree of the divisor. So, here the quotient is $2x - 1$ and the remainder is 3. Also, $(2x - 1)(x + 2) = 2x^2 + 3x - 2 + 3 = 2x^2 + 3x + 1$

$\Rightarrow$Dividend= Divisor x Quotient + Remainder

Example: Divide 7: $3x^3 + x^2 + 2x + 5$ by $1 + 2x + x^2$

$1 + 2x + x^2$	$3x^3 + x^2 + 2x + 5$	$3x - 5$
	$3x^3 + 6x^2 + 3x$	
	$-5x^2 - x + 5$ $-5x^2 - 10x - 5$	
	$9x + 10$	

We first arrange the terms of the dividend and the divisor in the decreasing order of their degrees. Recall that arranging the terms in this order is called writing the polynomials in standard form. In this example, the dividend is already in standard form, and the divisor, in standard form, is $x^2 + 2x + 1$

$(x^2 + 2x + 1)(3x - 5) + (9x + 10) = 3x^3 + x^2 + 2x + 5$

$\Rightarrow$Dividend= Divisor x Quotient + Remainder

If $p(x)$ and $g(x)$ are any two polynomials and $g(x) \neq 0$ then,

$p(x) = g(x) \cdot q(x) + r(x)$

$q(x)$ - Quotient and $r(x)$ - remainder

Here, $r(x) = 0$ or the degree of $r(x) <$ the degree of $g(x)$

This is known as The Division Algorithm for polynomials

Example 8: divide $3x^2 - x^3 - 3x + 5$ by $x - 1 - x^2$ and verify the division algorithm.

Note that To carry out division, we first write both the dividend and divisor in decreasing orders of their degrees. So, dividend $= -x^3 + 3x^2 - 3x + 5$ and divisor $= -x^2 + x - 1$.

$-x^2 + x - 1$	$-x^3 + 3x^2 - 3x + 5$	$x - 2$
	$-x3 + x^2 - x$	
	$2x^2 - 2x + 5$ $2x^2 - 2x + 2$	
	3	

$\therefore$ Quotient $= x - 2$, Remainder $= 3$

Divisor x Quotient + Remainder = $(-x^2 + x - 1)(x - 2) + 3 = -x^3 + x^2 - x + 2x^2 - 2x + 2 + 3$
$= -x^3 + 3x^2 - 3x + 5$ = Dividend. Hence, the division algorithm is verifyied.

Example 9 : Find all the zeroes of $2x^4 - 3x^3 - 3x^2 + 6x - 2$, if you know that two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$

Solution: Since two zeroes are $\sqrt{2}$ and $-\sqrt{2}$

$(x - \sqrt{2})(x + \sqrt{2}) = x^2 - 2$ are the factors of the polynomial

$x^2 - 2$	$2x^4 - 3x^3 - 3x^2 + 6x - 2$	$2x^2 - 3x + 1$
	$-2x^4 + \quad 4x^2$	
	$-3x^3 + x^2 + 6x - 2$	
	$-3x^3 \quad + 6x$	
	$+ x^2 \quad - 2$	
	$+ x^2 \quad - 2$	
	0	

Now, divide the polynomial by $x^2 - 2$

Factorise the Quotient = $2x^2 - 3x + 1$

$2x^2 - 2x - x + 1 = 2x(x - 1) - 1(2x - 1)$

$= (2x - 1)(x - 1)$

$\Rightarrow x = \frac{1}{2}$, $x = 1$ are the zeros

$\therefore$ All 4 zeros are $\sqrt{2}$, $-\sqrt{2}$, $\frac{1}{2}$, and 1

Exercise 9.3

1. **Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following :**

(i) $p(x) = x^3 - 3x^2 + 5x - 3$ $g(x) = x^2 - 2$ (ii) $p(x) = x^4 - 3x^2 + 4x + 5$ $g(x) = x^2 + 1 - x$

(iii) $p(x) = x^4 - 5x + 6$ $g(x) = 2 - x^2$

(i) $p(x) = x^3 - 3x^2 + 5x - 3$ $g(x) = x^2 - 2$

$x^2 - 2$	$x^3 - 3x^2 + 5x - 3$	$x - 3$
	$x^3 - 0 - 2x$	
	$-3x^2 + 7x - 3$	
	$-3x^2 + 0 + 6$	
	$+7x - 9$	

Quotient = $x - 3$; remainder = $7x - 9$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5$ $g(x) = x^2 + 1 - x$

$x^2 - x + 1$	$x^4 + 0.x^3 - 3x^2 + 4x + 5$	$x^2 + x - 3$
	$x^4 - x^3 + x^2$	
	$x^3 - 4x^2 + 4x$	
	$x^3 - x^2 + x$	
	$-3x^2 + 3x + 5$	
	$-3x^2 + 3x - 3$	
	8	

Quotient = $x^2 + x - 3$;remainder = 8

(iii) $p(x) = x^4 - 5x + 6$ $g(x) = 2 - x^2$

$-x^2+2$	$x^4+0x^3+0x^2-5x+6$	$-x^2-2$
	x^4+0-2x^2	
	$2x^2-5x+6$	
	$2x^2+0-4$	
	$-5x+10$	

Quotient $=-x^2-2$;remainder $= -5x+10$

2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) t^2-3, $2t^4+3t^3-2t^2-9t-12$ (ii) x^2+3x+1, $3x^4+5x^3-7x^2+2x+2$

(iii) x^3-3x+1, $x^5-4x^3+x^2+3x+1$

(i) t^2-3 $2t^4+3t^3-2t^2-9t-12$

t^2-3	$2t^4+3t^3-2t^2-9t-12$	$2t^2+3t+4$
	$2t^4+0-6t^2$	
	$+3t^3+4t^2-9t$	
	$+3t^3+0-9t$	
	$+4t^2+0-12$	
	$+4t^2+0-12$	
	0	

Remainder is Zero. Therefore first polynomial is the factor of the second polynomial.

(ii) x^2+3x+1 $3x^4+5x^3-7x^2+2x+2$

x^2+3x+1	$3x^4+5x^3-7x^2+2x+2$	$3x^2-4x+2$
	$3x^4+9x^3+3x^2$	
	$-4x^3-10x^2+2x$	
	$-4x^3-12x^2-4x$	
	$+2x^2+6x+2$	
	$+2x^2+6x+2$	
	0	

Remainder is Zero. Therefore first polynomial is the factor of the second polynomial.

(iii) x^3-3x+1 $x^5-4x^3+x^2+3x+1$

x^3-3x+1	$x^5-4x^3+x^2+3x+1$	x^2-1
	$x^5-3x^3+x^2$	
	$-x^3+0+3x+1$	
	$-x^3+0+3x-1$	
	2	

Remainder is 2 Therefore first polynomial is not the factor of the second polynomial.

3. **Obtain all other zeroes of $3x^4+6x^3-2x^2-10x-5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$**

$\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are the zeros of $3x^4+6x^3-2x^2-10x-5$

$\therefore \left(x - \sqrt{\frac{5}{3}}\right)$ and $\left(x + \sqrt{\frac{5}{3}}\right)$ are the factors of the polynomial.

$\Rightarrow \left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = x^2 - \frac{5}{3}$. Dividing the polynomial by $x^2 - \frac{5}{3}$

$x^2 - \frac{5}{3}$	$3x^4 + 6x^3 - 2x^2 - 10x - 5$	$3x^2 + 6x + 3$
	$3x^4 + 0 - 5x^2$	
	$+ 6x^3 + 3x^2 - 10x$	
	$+ 6x^3 + 0 - 10x$	
	$+3x^2 + 0 - 5$	
	$+3x^2 + 0 - 5$	
	0	

$3x^2 + 6x + 3 = 3(x^2 + 2x + 1)$

By Factorising $(x^2 + 2x + 1)$

$\Rightarrow x(x + 1) + 1(x + 1)$

$= (x + 1)(x + 1)$

Therefore The factors of $3x^4+6x^3-2x^2-10x-5$ are

$3\left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right)(x + 1)(x + 1)$

Therefore All the Zeros of the polynomials are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, -1$ and -1

4. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find g(x).

Dividend $P(x) = x^3 - 3x^2 + x + 2$; Divisor $g(x) = ?$;

Quotient $q(x) = x - 2$; Remainder $r(x) = -2x + 4$

$P(x) = g(x).q(x) + r(x)$

$x^3 - 3x^2 + x + 2 = g(x).(x - 2) + (-2x + 4)$

$\Rightarrow g(x).(x - 2) = x^3 - 3x^2 + x + 2 - (-2x + 4) \Rightarrow g(x).(x - 2) = x^3 - 3x^2 + x + 2 + 2x - 4$

$\Rightarrow g(x).(x - 2) = x^3 - 3x^2 + 3x - 2 \Rightarrow g(x) = \frac{x^3 - 3x^2 + 3x - 2}{x - 2}$

$x - 2$	$x^3 - 3x^2 + 3x - 2$	$x^2 - x + 1$
	$x^3 - 2x^2$	
	$- x^2 + 3x$	
	$- x^2 + 2x$	
	$+ x - 2$	
	$+ x - 2$	
	0	

$\therefore g(x) = x^2 - x + 1$

5. Give examples of polynomials p(x), g(x), q(x) and r(x), which satisfy the division algorithm and (i) degp(x) = degq(x) (ii) deg(x) = degr(x) (iii) deg r(x) = 0

(i) $p(x) = 6x^2 + 2x + 2$

$g(x) = 2$; $q(x) = 3x^2 + x + 1$

$r(x) = 0 \Rightarrow \deg p(x) = \deg q(x) = 2$

Verifying Division Algorithm,

$g(x) \times q(x) + r(x) = 2(3x^2 + x + 1) + 0$

$g(x) \times q(x) + r(x) = 6x^2 + 2x + 2 = P(x)$

$\Rightarrow p(x) = g(x) \times q(x) + r(x)$

$\therefore$ It is satisfying divison algorithm is verified.

(ii) $p(x) = x^3 + x$

$g(x) = x^2$; $q(x) = x$ and $r(x) = x$; $\deg g(x) = \deg r(x) = 1$

Verifying Division Algorithm,

$g(x) \times q(x) + r(x) = (x^2) \times x + x$

$g(x) \times q(x) + r(x) = x^3 + x = p(x) \quad \Rightarrow p(x) = g(x) \times q(x) + r(x)$

∴ It is satisfying divison algorithm is verified.

(iii) $p(x) = x^3 + 1$

$g(x) = x^2$; $q(x) = x$ and $r(x) = 1$; deg $r(x) = 0$

Verifying Division Algorithm

$g(x) \times q(x) + r(x) = (x^2) \times x + 1$

$\Rightarrow g(x) \times q(x) + r(x) = x^3 + 1 = P(x) \quad \Rightarrow p(x) = g(x) \times q(x) + r(x)$

∴ It is satisfying divison algorithm is verified.

Summary:

1. Polynomials of degrees 1, 2 and 3 are called linear, quadratic and cubic polynomials respectively.
2. A quadratic polynomial in x with real coefficients is of the form $ax^2 + bx + c$, where a, b, c are real numbers with a 0
3. The zeroes of a polynomial p(x) are precisely the x - coordinates of the points, where the graph of $y = p(x)$ intersects the x - axis
4. A quadratic polynomial can have at most 2 zeroes and a cubic polynomial can have at most 3 zeroes.
5. If α and β are the zeros of polynomial $ax^2 + bx + c$ then, $\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$
6. If α, β and γ are the zeros of $ax^3 + bx^2 + cx + d$ then,

$$\alpha + \beta + \gamma = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{-d}{a}$$

6. The division algorithm states that given any polynomial p(x) and any non-zero polynomial g(x), there are polynomials q(x) and r(x) such that $p(x) = g(x).\ q(x) + r(x)$ Here, $r(x) = 0$ or deg $r(x) <$ deg $g(x)$

10 Quadratic Equations

When we equate this polynomial to zero, we get a quadratic equation.

Any equation of the form p(x) = 0, where p(x) is a polynomial of degree 2, is a quadratic equation.

Standard form of quadratic equations:

$ax^2 + bx + c = 0$, Where $a \neq 0$

The features of quadratic equations:

- The quadratic equations has one variable
- The hieghest power of the variable is 2
- Standard form of quadratic equation: $ax^2 + bx + c = 0$,

Adfected quadratic equations : In a quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, $b \neq 0$ then it is called adfected quadratic equations.

Then, $x^2 - 3x - 5 = 0$, $x^2 + 5x + 6 = 0$, $x + \frac{1}{x} = 5$, $(2x - 5)^2 = 81$

Pure Quadratic equations : The quadrtic equations where a ≠ 0, b = 0 is called pure quadratic equations.

The standard form of pure quadratic equation: $ax^2 + c = 0$ $[a \neq 0]$

Example 1 : Represent the following situations mathematically:

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs 750. We would like to find out the number of toys produced on that day.

(i) Let the number of marbles with Jhon be 'x'

Then the number of marbles with Jivanthi = 45 – x [∵Total number of marbles 45]

The number of marbles left with John, when he lost 5 marbles = x – 5

The number of marbles left with Jivanti, when she lost 5 marbles = 45 – x – 5 = 40 – x

∴ Their products = 124

$(x - 5)(40 - x) = 124$

$\Rightarrow 40x - x^2 - 200 + 5x = 124 \Rightarrow -x^2 + 45x - 200 = 124$

$\Rightarrow -x^2 + 45x - 324 = 0 \Rightarrow x^2 - 45x + 324 = 0$

Therefore, the number of marbles John had, satisfies the quadratic equation $x^2 - 45x + 324 = 0$ which is the required representation of the problem mathematically.

(ii) Let the number of toys produced on that day be x.

Therefore, the cost of production (in rupees) of each toy that day = 55 – x

So, the total cost of production (in rupees) that day = x (55 – x)

∴ $x(55 - x) = 750$

$\Rightarrow 55x - x^2 = 750 \Rightarrow -x^2 + 55x - 750 = 0 \Rightarrow x^2 - 55x + 750 = 0$

∴ the number of toys produced that day satisfies the quadratic equation $x^2 - 55x - 750 = 0$ which is the required representation of the problem mathematically.

Example 2 : Check whether the following are quadratic equations:

(i) $(x-2)^2+1=2x-3$ (ii) $x(x+1)+8=(x+2)(x-2)$

(iii) $x(2x+3)=x^2+1$ (iv) $(x+2)^3=x^3-4$

(i) $(x-2)^2+1=2x-3$

$x^2-4x+4+1=2x-3 \Rightarrow x^2-4x-2x+5+3=0 \Rightarrow x^2-6x+8=0$

This is in the form of $ax^2+bx+c=0$

Therefore the given equation is quadratic equation.

(ii) $x(x+1)+8=(x+2)(x-2)$

$x^2+x+8=x^2-4 \Rightarrow x^2-x^2+x+8+4=0 \Rightarrow x+12=0$

This is not in the form of $ax^2+bx+c=0$

Therefore the given equation is not a quadratic equation.

(iii) $x(2x+3)=x^2+1$

$2x^2+3x=x^2+1 \Rightarrow 2x^2-x^2+3x-1=0 \Rightarrow x^2+3x-1=0$

This is in the form of $ax^2+bx+c=0$

Therefore the given equation is quadratic equation.

(iv) $(x+2)^3=x^3-4$

$x^3+2^3+3(x)(2)^2+3x^2(2)=x^3-4$

$x^3+8+12x+6x^2=x^3-4 \Rightarrow x^3-x^3+6x^2+12x+8+4=0$

$\Rightarrow 6x^2+12x+12=0 \div 6 \Rightarrow x^2+2x+2=0$

This is in the form of $ax^2+bx+c=0$

Therefore the given equation is quadratic equation.

Exercise 10.1

1. Check whether the following are quadratic equations :

(i) $(x+1)^2=2(x-3)$ (ii) $x^2-2x=(-2)(3-x)$ (iii) $(x-2)(x+1)=(x-1)(x+3)$

(iv) $(x-3)(2x+1)=x(x+5)$ (v) $(2x-1)(x-3)=(x+5)(x-1)$ (vi) $x^2+3x+1=(x-2)^2$

(vii) $(x+2)^3=2x(x^2-1)$ (viii) $x^3-4x^2-x+1=(x-2)^3$

(i) $(x+1)^2=2(x-3)$

$x^2+2x+1=2x-6 \Rightarrow x^2+2x-2x+1+6=0 \Rightarrow x^2+7=0$

This is in the form of $ax^2+bx+c=0$

Therefore the given equation is quadratic equation.

(ii) $x^2-2x=(-2)(3-x)$

$x^2-2x=-6+2x \Rightarrow x^2-2x-2x+6=0 \Rightarrow x^2-4x+6=0$

This is in the form of $ax^2+bx+c=0$

Therefore the given equation is quadratic equation.

(iii) $(x-2)(x+1)=(x-1)(x+3)$

$x^2+x-2x-2=x^2+3x-x-3 \Rightarrow x^2-x-2=x^2+2x-3$

$\Rightarrow x^2-x^2-x-2x-2+3=0 \Rightarrow -3x+3=0 \times -1 \Rightarrow 3x-1=0$

This is not in the form of $ax^2+bx+c=0$

Therefore the given equation is not a quadratic equation.

(iv) $(x-3)(2x+1)=x(x+5)$

$2x^2+x-6x-3=x^2+5x \Rightarrow 2x^2-5x-3=x^2+5x$

$\Rightarrow 2x^2-x^2-5x-5x-3=0 \Rightarrow x^2-10x-3=0$

This is in the form of $ax^2+bx+c=0$

Therefore the given equation is quadratic equation.

(v) $(2x-1)(x-3)=(x+5)(x-1)$

$2x^2-6x-x+3=x^2-x+5x-5 \Rightarrow 2x^2-7x+3=x^2+4x-5$

$\Rightarrow 2x^2-x^2-7x-4x+3+5=0 \Rightarrow x^2-11x+8=0$

This is in the form of $ax^2+bx+c=0$

Therefore the given equation is quadratic equation.

(vi) $x^2+3x+1=(x-2)^2$

$x^2+3x+1=x^2-2(x)(2)+2^2 \Rightarrow x^2-x^2+3x+4x+1-4=0$

$7x-3=0$

This is not in the form of $ax^2+bx+c=0$

Therefore the given equation is not a quadratic equation.

(vii) $(x+2)^3=2x(x^2-1)$

$x^3+2^3+3(x)(2)^2+3x^2(2)=2x^3-2x \Rightarrow x^3+8+12x+6x^2=2x^3-2x$

$\Rightarrow x^3-2x^3+6x^2+12x+2x+8=0 \Rightarrow -x^3+6x^2+14x+8=0\times -1$

$\Rightarrow x^3-6x^2-14x-8=0$

This is not in the form of $ax^2+bx+c=0$

Therefore the given equation is not a quadratic equation.

(viii) $x^3-4x^2-x+1=(x-2)^3$

$x^3-4x^2-x+1=x^3-2^3+3(x)(2)^2-3x^2(2)$

$\Rightarrow x^3-4x^2-x+1=x^3-8+12x-6x^2$

$\Rightarrow x^3-x^3-4x^2+6x^2-x-12x+1+8=0 \Rightarrow 2x^2-13x+9=0$

This is in the form of $ax^2+bx+c=0$

Therefore the given equation is quadratic equation.

2. **Represent the following situations in the form of quadratic equations :**

(i) The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

Let breadth $b=x$ m $\Rightarrow$ Length $l=(2x+1)$m

Area of the rectangle $=l\times b \Rightarrow 528=x(2x+1) \Rightarrow 528=2x^2+x$

$\Rightarrow 2x^2+x-528=0$

(ii) The product of two consecutive positive integers is 306. We need to find the integers.

Let two consecutive integers be x and $(x+1)$; Their products $=306$

$\Rightarrow x(x+1)=306 \Rightarrow x^2+x-306=0$

(iii)Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

Let the present age of Rohan $=x$; The present age of his mother $=x+26$

After 3 Rohan's age $=x+3$

After 3 years his mothers age $=x+26+3=x+29$

Product of their ages after 3 years $=360$

$\therefore (x+3)(x+29)=360 \Rightarrow x^2+29x+3x+87=360$

$\Rightarrow x^2+32x-273=0$

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Let the speed of the train $=x\ km/h$

The time taken to travel 480 km $=\frac{480}{x}\ hrs$

Reducing speed by 8 km/h, the speed of the train $=(x-8)km/h$

Therefore the time taken to travel 480 km $= \left(\frac{480}{x-8}\right) hrs$

$\Rightarrow \frac{480}{x} + 3 = \frac{480}{x-8} \Rightarrow 480(x-8) + 3x(x-8) = 480x$

$\Rightarrow 480x - 3840 + 3x^2 - 24x = 480x \Rightarrow 3840 + 3x^2 - 24x = 0$

$\Rightarrow 3x^2 - 24x + 3840 = 0$

$\Rightarrow x^2 - 8x + 1280 = 0$

10.3 Solution of a Quadratic Equation by Factorisation

Note: The zeros of the quadratic polynomial $ax^2 + bx + c$ and the roots of the quadratic equation are the same.

Example 3 : Find the roots of the equation $2x^2 - 5x + 3 = 0$, by factorization.

$2x^2 - 5x + 3 = 0$

$\Rightarrow 2x^2 - 2x - 3x + 3 = 0$

$\Rightarrow 2x(x-1) - 3(x-1) =$

$\Rightarrow (x-1)(2x-3) = 0$

$\Rightarrow x - 1 = 0, \quad 2x - 3 = 0$

$\Rightarrow x = 1, \ 2x = 3$

$x = 1, \ x = \frac{3}{2}$

First term = $\mathbf{2x^2}$, Last term = $\mathbf{+3}$
Their product = $+\mathbf{6x^2}$
The middle term = $\mathbf{-5x}$
Divide middle term such that product = $+\mathbf{6x^2}$ and their sum
$\mathbf{-5x} \Rightarrow -5x = -2x - 3x$

Example 4: Find the roots the equation $6x^2 - x - 2 = 0$

$6x^2 - x - 2 = 0$

$6x^2 - 4x + 3x - 2 = 0$

$2x(3x-2) + 1(3x-2) = 0$

$(2x+1)(3x-2) = 0$

$2x + 1 = 0, 3x - 2 =$

$2x = -1, 3x = 2$

$\Rightarrow x = \frac{-1}{2}, x = \frac{2}{3}$

First term = $6x^2$, Last term = -2
Their product = $-12x^2$
The middle term = $-x$
Divide middle term such that product = $-12x^2$ and sum $-x$
$\Rightarrow -x = -4x + 3x$

Example 5:Find the roots the equation $3x^2 - 2\sqrt{6}\,x + 2 = 0$

$\mathbf{3x^2 - 2\sqrt{6}\,x + 2 = 0}$

$3x^2 - \sqrt{6}\,x - \sqrt{6}\,x + 2 = 0$

$(\sqrt{3})^2 x^2 - \sqrt{2}.\sqrt{3}\,x - \sqrt{2}.\sqrt{3}\,x + (\sqrt{2})^2 = 0$

$\sqrt{3}\,x(\sqrt{3}\,x - \sqrt{2}) - \sqrt{2}(\sqrt{3}\,x - \sqrt{2}) = 0$

$(\sqrt{3}\,x - \sqrt{2})(\sqrt{3}\,x - \sqrt{2}) = 0$

$(\sqrt{3}\,x - \sqrt{2}) = 0, \ (\sqrt{3}\,x - \sqrt{2}) = 0$

$\sqrt{3}\,x = \sqrt{2}, \ \sqrt{3}\,x = \sqrt{2} \Rightarrow x = \sqrt{\frac{2}{3}}, \ x = \sqrt{\frac{2}{3}}$

First term = $3x^2$, Last term = $+2$
Their product = $6x^2$
The middle term = $-2\sqrt{6}\,x$
Divide middle term such that product = $6x^2$ and sum $-2\sqrt{6}\,x$
$\Rightarrow -2\sqrt{6}\,x = \sqrt{6}\,x - \sqrt{6}\,x$

Example 6 : Find the dimensions of the prayer hall discussed in Section 10.1.

$2x^2 + x - 300 = 0$

$2x^2 - 24x + 25x - 300 = 0$

$2x(x-12) + 25(x-12) = 0$

$(x-12)(2x+25) =$

$x - 12 = 0, \quad 2x + 25 = 0$

$x = 12, \ 2x = -25 \Rightarrow x = \frac{-25}{2} = -12.5$

Breadth $= x = 12\ m$

Length $= 2x + 1 = 2(12) + 1 = 24 + 1 = 25\ m$

First term = $2x^2$, Last term = -300
Their produ= $-600x^2$
The middle term = $+x$
Divide middle term such that product = $-600x^2$ and sum x
$\Rightarrow +x = -24x + 25x$

Exercise 10.2

1. Find the roots of the following quadratic equations by factorisation:

(i) $x^2 - 3x - 10 = 0$ (ii) $2x^2 + x - 6 = 0$ (iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

(iv) $2x^2 - x + \frac{1}{8} = 0$ (v) $100x^2 - 20x + 1 = 0$

(i) $x^2 - 3x - 10 = 0$

$x^2 - 5x + 2x - 10 = 0 \quad \Rightarrow x(x-5) + 2(x-5) = 0$

$\Rightarrow (x-5)(x+2) = 0 \quad \Rightarrow (x-5) = 0, (x+2) = 0$

$\Rightarrow x = 5, x = -2$

(ii) $2x^2 + x - 6 = 0$

$2x^2 + x - 6 = 0 \quad \Rightarrow 2x^2 + 4x - 3x - 6 = 0$

$\Rightarrow 2x(x+2) - 3(x+2) = 0 \quad \Rightarrow (x+2)(2x-3) = 0$

$\Rightarrow x + 2 = 0, 2x - 3 = 0$

$\Rightarrow x = -2, 2x = 3 \Rightarrow x = -2, x = \frac{3}{2}$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$

$\Rightarrow \sqrt{2}x(x + \sqrt{2}) + 5(x + \sqrt{2}) = 0 \quad \Rightarrow (\sqrt{2}x + 5)(x + \sqrt{2}) = 0$

$\Rightarrow \sqrt{2}x + 5 = 0, \ x + \sqrt{2} = 0 \Rightarrow \sqrt{2}x = -5, \ x = -\sqrt{2} \quad \Rightarrow x = \frac{-5}{\sqrt{2}}, \quad x = -\sqrt{2}$

(iv) $2x^2 - x + \frac{1}{8} = 0$

$16x^2 - 8x + 1 = 0$

$\Rightarrow 16x^2 - 4x - 4x + 1 = 0 \quad \Rightarrow 4x(4x-1) - 1(4x-1) = 0$

$\Rightarrow (4x-1)(4x-1) = 0 \Rightarrow 4x - 1 = 0, \ 4x - 1 = 0$

$\Rightarrow 4x = 1, 4x = 1 \Rightarrow x = \frac{1}{4}, \ x = \frac{1}{4}$

(v) $100x^2 - 20x + 1 = 0$

$100x^2 - 20x + 1 = 0$

$\Rightarrow 100x^2 - 10x - 10x + 1 = 0 \Rightarrow 10x(10x-1) - 1(10x-1) = 0$

$\Rightarrow (10x-1)(10x-1) = 0 \quad \Rightarrow 10x - 1 = 0, 10x - 1 = 0$

$\Rightarrow 10x = 1, 10x = 1 \Rightarrow x = \frac{1}{10}, \ x = \frac{1}{10}$

2. Solve the problems given in Example 1.

In example 1 we got the equations: (i) $x^2 - 45x + 324 = 0$ **and (ii)** $x^2 - 55x + 750 = 0$

(i) $x^2 - 45x + 324 = 0$

$x^2 - 36x - 9x + 324 = 0 \Rightarrow x(x-36) - 9(x-36) = 0$

$\Rightarrow (x-36)(x-9) = 0 \Rightarrow (x-36) = 0, \quad (x-9) = 0$

$\Rightarrow x = 36, \quad x = 9$

The marbles with John $= x = 36$ and the marbles with Jevan $= 45 - x = 45 - 36 = 9$

Or The marbles with John $x = 9$ and the marbles with Jevan: $45 - x = 45 - 9 = 36$

(ii) $x^2 - 55x + 750 = 0$

$x^2 - 25x - 30x + 750 = 0 \Rightarrow x(x-25) - 30(x-25) = 0$

$\Rightarrow (x-25)(x-30) = 0 \Rightarrow (x-25) = 0, \quad (x-30) = 0$

$\Rightarrow x = 25, \quad x = 30$

The number of toys are 25 or 30

3. Find two numbers whose sum is 27 and product is 182.

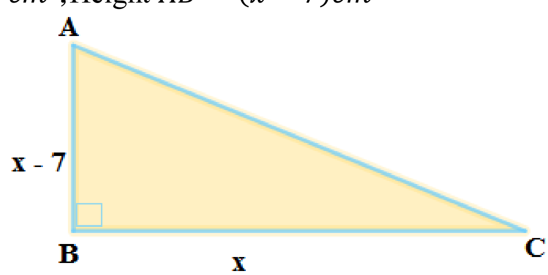
Let the first number $= x$ then second number $= 27 - x$

Their product $= 182$

$\therefore x(27-x)=182 \Rightarrow 27x-x^2=182$
$-x^2+27x-182=0 \times -1$
$\mathbf{x^2-27x+182=0}$
$x^2-13x-14x+182=0$
$\Rightarrow x(x-13)-14(x-13)=0 \Rightarrow (x-13)(x-14)=0$
$\Rightarrow (x-13)=0, \quad (x-14)=0 \Rightarrow x=13, x=14$
If first number $=13$ then the second number $=27-13=14$
If first number $=14$ then the second number$=27-14=13$
Therefore the numbers are 13 and 14

4. Find two consecutive positive integers, sum of whose squares is 365.
Let the positive numbsr be x and the concecutiveinteger $=x+1$
$x^2+(x+1)^2=365 \Rightarrow x^2+x^2+2x+1=35$
$\Rightarrow 2x^2+2x+1-365=0 \Rightarrow 2x^2+2x-364=0 \div 2$
$\mathbf{x^2+x-182=0}$
$x^2+14x-13x-182=0 \Rightarrow x(x+14)-13(x+14)=0$
$\Rightarrow (x+14)(x-13)=0 \Rightarrow (x+14)=0, (x-13)=0$
$\Rightarrow x=-14, x=13$
$\therefore x+1=13+1=14$
Two consecutive positive integers13, 14 ಆಗಿವೆ.

5. **The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.**
Let the base of the right angle triangle $BC=x\ cm$;Height $AB=(x-7)cm$



According to Pythagoras theorem,
$BC^2+AB^2=AC^2$
$x^2+(x-7)^2=13^2$
$x^2+x^2+7^2-2(x)(7)=169$
$2x^2-14x+49-169=0$
$2x^2-14x-120=0 \div 2$
$\mathbf{x^2-7x-60=0}$
$x^2-12x+5x-60=0 \Rightarrow x(x-12)+5(x-12)=0$
$\Rightarrow (x-12)(x+5)=0 \Rightarrow (x-12)=0, \quad (x+5)=0$
$\Rightarrow x=12, \ x=-5$
The base of $BC=x=12\ cm$
Height $AB=(x-7)=12-7=5cm$

6. **A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was ` 90, find the number of articles produced and the cost of each article.**
Let the number of pots $=x$; The cost $=Rs(2x+3)$

$\therefore x(2x+3)=90$
$\mathbf{2x^2+3x-90=0}$
$2x^2+15x-12x-90=0 \Rightarrow x(2x+15)-6(2x+15)=0$
$\Rightarrow (2x+15)(x-6)=0 \Rightarrow (2x+15)=0, (x-6)=0$
$\Rightarrow 2x=-15, \ x=6 \Rightarrow x=\frac{-15}{2}, x=6$
Therefore number of pots $=x=6$; Cost $=(2x+3)=2(6)+3=s12+3=R15$

10.4 Solution of a Quadratic Equation by Completing the Square

Example: Solve the equation $3x^2 - 5x + 2 = 0$ by completing the square.

$3x^2 - 5x + 2 = 0 \qquad \times 3$

$\Rightarrow 9x^2 - 15x + 6 = 0 \Rightarrow 9x^2 - 15x = -6$

Add b^2 to both the sides

$(3x)^2 - 2(3x)\left(\frac{5}{2}\right) + \left(\frac{5}{2}\right)^2 = -6 + \left(\frac{5}{2}\right)^2$

$\left(3x - \frac{5}{2}\right)^2 = -6 + \frac{25}{4} \Rightarrow \left(3x - \frac{5}{2}\right)^2 = \frac{-24+25}{4}$

$\Rightarrow \left(3x - \frac{5}{2}\right)^2 = \frac{1}{4} \Rightarrow \left(3x - \frac{5}{2}\right) = \pm\sqrt{\frac{1}{4}}$

$\Rightarrow \left(3x - \frac{5}{2}\right) = \pm\frac{1}{2} \Rightarrow 3x = \pm\frac{1}{2} + \frac{5}{2} \Rightarrow 3x = \frac{\pm 1 + 5}{2} \Rightarrow x = \frac{1+5}{6},\ x = \frac{-1+5}{6}$

$\Rightarrow x = \frac{6}{6},\ x = \frac{4}{6} \Rightarrow x = 1,\ x = \frac{2}{3}$

$2ab = 15x$
$2(3x)b = 15x$
$b = \frac{15x}{6x} = \frac{5}{2}$
$b^2 = \left(\frac{5}{2}\right)^2$

Example 7 : Solve the equation given in Example 3 by the method of completing the the square: $2x^2 - 5x + 3 = 0$

$2x^2 - 5x + 3 = 0 \qquad \times 2$

$4x^2 - 10x + 6 = 0 \Rightarrow 4x^2 - 10x = -6$

Add b^2 to both the sides

$(2x)^2 - 2(2x)\left(\frac{5}{2}\right) + \left(\frac{5}{2}\right)^2 = -6 + \left(\frac{5}{2}\right)^2$

$\Rightarrow \left(2x - \frac{5}{2}\right)^2 = -6 + \frac{25}{4} \Rightarrow \left(2x - \frac{5}{2}\right)^2 = \frac{-24+25}{4}$

$\Rightarrow \left(2x - \frac{5}{2}\right)^2 = \frac{1}{4} \Rightarrow \left(2x - \frac{5}{2}\right) = \pm\sqrt{\frac{1}{4}}$

$\Rightarrow \left(2x - \frac{5}{2}\right) = \pm\frac{1}{2} \Rightarrow 2x = \pm\frac{1}{2} + \frac{5}{2} \Rightarrow 2x = \frac{\pm 1+1}{2} \Rightarrow x = \frac{1+5}{4},\ x = \frac{-1+5}{4}$

$\Rightarrow x = \frac{6}{4},\ x = \frac{4}{4} \Rightarrow x = \frac{3}{2},\ x = 1$

$2ab = 10x$
$2(2x)b = 15x$
$b = \frac{10x}{4x} = \frac{5}{2}$
$b^2 = \left(\frac{5}{2}\right)^2$

Example 8 : Find the roots of the equation $5x^2 - 6x - 2 = 0$ by the method of completing the square.

$5x^2 - 6x - 2 = 0 \qquad x\ 5$

$25x^2 - 30x - 10 = 0 \Rightarrow 25x^2 - 30x = 10$

Add b^2 to both the sides

$25x^2 - 30x + (3)^2 = 10 + (3)^2$

$\Rightarrow (5x)^2 - 2(5x)(3) + (3)^2 = 10 + 9 \Rightarrow (5x - 3)^2 = 19$

$\Rightarrow (5x - 3) = \pm\sqrt{19} \Rightarrow 5x = 3 \pm \sqrt{19}$

$\Rightarrow x = \frac{3\pm\sqrt{19}}{5} \Rightarrow x = \frac{3+\sqrt{19}}{5},\quad x = \frac{3-\sqrt{19}}{5}$

$2ab = 30x$
$2(5x)b = 30x$
$b = \frac{30x}{10x} = 3$
$b^2 = (3)^2$

Example 9 : Find the roots of $4x^2 + 3x + 5 = 0$ by the method of completing the square.

$4x^2 + 3x + 5 = 0 \quad \Rightarrow 4x^2 + 3x = -5$

Add b^2 to both the sides

$(2x)^2 - 2(2x)\left(\frac{3}{4}\right) + \left(\frac{3}{4}\right)^2 = -5 + \left(\frac{3}{4}\right)^2$

$\Rightarrow \left(2x - \frac{3}{4}\right)^2 = -5 + \frac{9}{16} \Rightarrow \left(2x - \frac{3}{4}\right)^2 = \frac{-80+9}{16}$

$2ab = 3x$
$2(2x)b = 3x$
$b = \frac{3x}{4x} = \frac{3}{4}$
$b^2 = \left(\frac{3}{4}\right)^2$

$\Rightarrow\left(2x-\frac{3}{4}\right)^2=\frac{-71}{16}<0$ ⇒There is no roots for this quadratic equation . The roots are imaginary.

Solving the quadratic equations using formula:

Find the roots of the quadratic equation $ax^2+bx+c=0$ by completing the square.

$ax^2+bx=-c$ [multiply the equation by 4a]

$4a^2x^2+4abx=-4ac$ [Add b^2 to both the sides]

$4a^2x^2+4abx+b^2=-4ac+b^2$

$\Rightarrow(2ax)^2+2(2ax)(b)+b^2=b^2-4ac\Rightarrow(2ax+b)^2=b^2-4ac$

$\Rightarrow 2ax+b=\pm\sqrt{b^2-4ac} \Rightarrow 2ax=-b\pm\sqrt{b^2-4ac} \Rightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Roots are: $x=\frac{-b+\sqrt{b^2-4ac}}{2a}$, $x=\frac{-b-\sqrt{b^2-4ac}}{2a}$

Example 10 : Solve Q. 2(i) of Exercise 10.1 by using the quadratic formula

In Q.No.2(i) of exercise 10.i we got the equation $2x^2+x-528=0$

This is in the form of $ax^2+bx+c=0$

$a=2,\quad b=1,\quad c=-528$

Roots are $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(1)\pm\sqrt{(1)^2-4(2)(-528)}}{2(2)}\Rightarrow x=\frac{-1\pm\sqrt{1+4224}}{4}$

$\Rightarrow x=\frac{-1\pm\sqrt{4225}}{4}\Rightarrow x=\frac{-1\pm65}{4}$

$x=\frac{-1+65}{4}$ or $x=\frac{-1-65}{4}$

$x=\frac{64}{4}$ or $x=\frac{-66}{4}$

$x=16$ or $x=-\frac{33}{2}$

⇒Breadth of the site = 16m and the length = 2x16 + 1 = 32 + 1 = 33m

Example 11:Find two consecutive odd positive integers, sum of whose squares is 290.

Let the consecutive odd numbers be x and $x+2$

$x^2+(x+2)^2=290$

$x^2+x^2+2^2+2(x)(2)=290$

$2x^2+4x+4-290=0$

$2x^2+4x-286=0$ $\div 2$

$x^2+2x-143=0$ this is in the form of $ax^2+bx+c=0$

$a=1,\quad b=2,\quad c=-143$

Roots are $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(2)\pm\sqrt{(2)^2-4(1)(-143)}}{2(1)}\Rightarrow x=\frac{-2\pm\sqrt{4+572}}{2}$

$\Rightarrow x=\frac{-2\pm\sqrt{576}}{2}\Rightarrow x=\frac{-2\pm24}{2}$

$x=\frac{-2+24}{2}$, $x=\frac{-2-24}{2}$

$x=\frac{22}{2}$, $x=\frac{-26}{2}$

$x=11,\quad x=-13$

Therefore the consecutive odd numbers are 11 and 13

Example 12 : A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m (see Fig. 10.3). Find its length and

Solution: The breadth of the rectangular park be x m and the length = $(x+3)$ m

Area = $x(x + 3)m^2 = (x^2 + 3x)m^2$.

Now, the base of the isosceles triangle = x m

Therefore the area = $\frac{1}{2} \times x \times 12 = 6x$ m

According to question, $x^2 + 3x = 6x + 4$

∴ $\mathbf{x^2 - 3x - 4 = 0}$ this is in the form of $ax^2 + bx + c = 0$

$a = 1, \quad b = -3, \quad c = -4$

The roots are $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-4)}}{2(1)} \Rightarrow x = \frac{3 \pm \sqrt{9+16}}{2}$

$\Rightarrow x = \frac{3 \pm \sqrt{25}}{2} \Rightarrow x = \frac{3 \pm 5}{2}$

$x = \frac{3+5}{2}, \ x = \frac{3-5}{2}$

$x = \frac{8}{2}, x = \frac{-2}{2}$

$x = 4, x = -1$

∴ The breadth = x = 4m and the length = x + 3 = 4 + 3 = 7m.

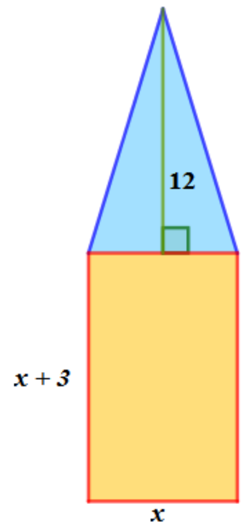


Fig 10.3

Example 13 : Find the roots of the following quadratic equations, if they exist, using the quadratic formula.(i) $3x^2 - 5x + 2 = 0$ (ii) $x^2 + 4x + 5 = 0$ (iii) $2x^2 - 2\sqrt{2}x + 1$

(i) $3x^2 - 5x + 2 = 0$ this is in the form of $ax^2 + bx + c = 0$

$a = 3, \quad b = -5, \quad c = +2$

Roots are: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-(-5) \pm \sqrt{(-5)^2-4(3)(2)}}{2(3)} \Rightarrow x = \frac{5 \pm \sqrt{25-24}}{6}$

$\Rightarrow x = \frac{5 \pm \sqrt{1}}{6} \Rightarrow x = \frac{5 \pm 1}{6}$

$x = \frac{6}{6}$ or $x = \frac{4}{6} \Rightarrow x = 1$ or $x = \frac{2}{3}$

(ii) $x^2 + 4x + 5 = 0$ this is in the form of $ax^2 + bx + c = 0$

$a = 1, \quad b = 4, \quad c = +5$

Roots are $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-(4) \pm \sqrt{(4)^2-4(1)(5)}}{2(1)} \Rightarrow x = \frac{-4 \pm \sqrt{16-20}}{2}$

$\Rightarrow x = \frac{-4 \pm \sqrt{-4}}{2} \Rightarrow x = \frac{-4 \pm 2\sqrt{-1}}{2} \Rightarrow$ Roots are not Real numbers.

(iii) $2x^2 - 2\sqrt{2}x + 1$ this is in the form of $ax^2 + bx + c = 0$

$a = 2, \quad b = -2\sqrt{2}, \quad c = +1$

roots are, $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-(-2\sqrt{2}) \pm \sqrt{(-2\sqrt{2})^2 - 4(2)(1)}}{2(2)}$

$\Rightarrow x = \frac{2\sqrt{2} \pm \sqrt{8-8}}{4} \Rightarrow x = \frac{2\sqrt{2} \pm \sqrt{0}}{4} \Rightarrow x = \frac{2\sqrt{2}}{4} \Rightarrow x = \frac{\sqrt{2}}{2} \Rightarrow \frac{1}{\sqrt{2}}$

Example 14:Find the roots of the following equations

(i) $x + \frac{1}{x} = 3, x \neq 0$ (ii) $\frac{1}{x} + \frac{1}{x-2} = 3 \quad x \neq 0, x \neq 2$

(i) $x + \frac{1}{x} = 3, x \neq 0$

$x + \frac{1}{x} = 3$ – Multiply both sides bu x

$x^2 + 1 = 3x \Rightarrow x^2 - 3x + 1 = 0$ this is in the form of $ax^2 + bx + c = 0$

$a = 1, \quad b = -3, \quad c = 1$

Roots are, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)} \Rightarrow x = \frac{3 \pm \sqrt{9-4}}{2} \Rightarrow x = \frac{3 \pm \sqrt{5}}{2} \Rightarrow x = \frac{3+\sqrt{5}}{2}, \; x = \frac{3-\sqrt{5}}{2}$

(ii) $\frac{1}{x} - \frac{1}{x-2} = 3 \Rightarrow \frac{x-2-x}{x(x-2)} = 3$

$\Rightarrow \frac{-2}{x^2-2x} = 3 \Rightarrow -2 = 3x^2 - 6x$

$3x^2 - 6x + 2 = 0$ this is in the form of $ax^2 + bx + c = 0$

$a = 3, \quad b = -6, \quad c = 2$

roots are, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(2)}}{2(3)}$

$\Rightarrow x = \frac{6 \pm \sqrt{36-24}}{6} \Rightarrow x = \frac{6 \pm \sqrt{12}}{6} \Rightarrow x = \frac{6 \pm \sqrt{4 \times 3}}{6} \Rightarrow x = \frac{6 \pm 2\sqrt{3}}{6}$

$\Rightarrow x = \frac{2(3 \pm \sqrt{3})}{6} \Rightarrow x = \frac{3 \pm \sqrt{3}}{3} \Rightarrow x = \frac{3+\sqrt{3}}{3}, \quad x = \frac{3-\sqrt{3}}{3}$

Example 15 : A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Speed of the stream: $= x$ km/h

Speed of the motor boat upstream $= (18 - x)$km/h

Speed of the motor boat downstram $= (18 + x)$km/h

Time taken to travel upsteram $= \frac{24}{18-x}$ hour

Time taken to travel down steram $= \frac{24}{18+x}$ hour

$\frac{24}{18-x} - \frac{24}{18+x} = 1,$.

$24(18 + x) - 24(18 - x) = 1(18 - x)(18 + x)$

$432 + 24x - 432 + 24x = 324 - x^2 \Rightarrow 48x = 324 - x^2$

$-x^2 + 324 - 48 = 0$ x (-1)

$x^2 + 48x - 324 = 0$ this is in the form of $ax^2 + bx + c = 0$

$a = 1, \quad b = 48, \quad c = -324$

Roots are, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(48) \pm \sqrt{(-48)^2 - 4(1)(-324)}}{2(1)}$

$\Rightarrow x = \frac{-48 \pm \sqrt{2304+1296}}{2} \Rightarrow x = \frac{-48 \pm \sqrt{3600}}{2} \Rightarrow x = \frac{-48 \pm 60}{2}$

$\Rightarrow x = \frac{-48+60}{2}, \quad x = \frac{-48-60}{2}$

$\Rightarrow x = \frac{12}{2}, \; x = \frac{-108}{2}$

$\Rightarrow x = 6, \; x = -54$

$\Rightarrow$Speed of the stream$= x = 6 \; km/h$

Exercise 10.3

1. **Find the roots of the following quadratic equations, if they exist, by the method of completing the square:.**

   (i) $2x^2 - 7x + 3 = 0$ (ii) $2x^2 + x - 4 = 0$(iii) $4x^2 + 4\sqrt{3}\,x + 3 = 0$ (iv) $2x^2 + x + 4 = 0$

(i) $2x^2 - 7x + 3 = 0$

$2x^2 - 7x = -3 \times 2$

$4x^2 - 14x = -6$

$4x^2 - 14x + \left(\frac{7}{2}\right)^2 = -6 + \left(\frac{7}{2}\right)^2$ [Add $\left(\frac{7}{2}\right)^2$ to both the sides]

$(2x)^2 - 2(2x)(\frac{7}{2}) + \left(\frac{7}{2}\right)^2 = -6 + \frac{49}{4}$

$\left(2x - \frac{7}{2}\right)^2 = \frac{-24+49}{4} \Rightarrow \left(2x - \frac{7}{2}\right) = \pm\sqrt{\frac{25}{4}}$

$\Rightarrow 2x - \frac{7}{2} = \pm\frac{5}{2} \Rightarrow 2x = \pm\frac{5}{2} + \frac{7}{2} \Rightarrow 2x = \frac{\pm5+7}{2} \Rightarrow x = \frac{\pm5+7}{4}$, $2x = 1$

$\Rightarrow x = \frac{5+7}{4}$, $x = \frac{-5+7}{4} \Rightarrow x = \frac{12}{4}$, $x = \frac{2}{4}$

$\Rightarrow x = 3$, $x = \frac{1}{2}$

$2ab = 14x$

$2(2x)b = 14x$

$b = \frac{14x}{4x} = \frac{7}{2}$

$b^2 = \left(\frac{7}{2}\right)^2$

(ii) $2x^2 + x - 4 = 0$

$2x^2 + x = 4$ $\times 2$

$4x^2 + 2x = 8$

$4x^2 + 2x + \left(\frac{1}{2}\right)^2 = 8 + \left(\frac{1}{2}\right)^2$ [Add $\left(\frac{1}{2}\right)^2$ to both the sides]

$(2x)^2 + 2(2x)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 = 8 + \frac{1}{4}$

$\left(2x + \frac{1}{2}\right)^2 = \frac{32+1}{4} \Rightarrow \left(2x + \frac{1}{2}\right) = \pm\sqrt{\frac{33}{4}}$

$\Rightarrow 2x + \frac{1}{2} = \pm\frac{\sqrt{33}}{2} \Rightarrow 2x = \pm\frac{\sqrt{33}}{2} + \frac{1}{2} \Rightarrow 2x = \frac{\pm\sqrt{33}-1}{2} \Rightarrow x = \frac{\pm\sqrt{33}-1}{4}$

$\Rightarrow x = \frac{\sqrt{33}-1}{4}$, $x = \frac{-\sqrt{33}-1}{4}$

$2ab = 2x$

$2(2x)b = 2x$

$b = \frac{2x}{4x} = \frac{1}{2}$

$b^2 = \left(\frac{1}{2}\right)^2$

(iii) $4x^2 + 4\sqrt{3}\,x + 3 = 0$

$4x^2 + 4\sqrt{3}\,x = -3$

$4x^2 + 4\sqrt{3}x + \left(\sqrt{3}\right)^2 = -3 + \left(\sqrt{3}\right)^2$[Add $\left(\sqrt{3}\right)^2$ to both the sides]

$(2x)^2 + 2(2x)(\sqrt{3}) + \left(\sqrt{3}\right)^2 = -3 + 3$

$\left(2x + \sqrt{3}\right)^2 = 0$

$\left(2x + \sqrt{3}\right) = 0$, $\left(2x + \sqrt{3}\right) = 0$

$2x = -\sqrt{3}$, $2x = -\sqrt{3} \Rightarrow x = \frac{-\sqrt{3}}{2}$, $x = \frac{-\sqrt{3}}{2}$

$2ab = \mathbf{4}\sqrt{3}x$

$2(2x)b = \mathbf{4}\sqrt{3}x$

$b = \frac{4\sqrt{3}x}{4x} = \sqrt{3}$

$b^2 = \left(\sqrt{3}\right)^2$

(iv) $2x^2 + x + 4 = 0$

$2x^2 + x = -4$ $\times 2$

$4x^2 + 2x = -8$

$(2x)^2 - 2(2x)\left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2 = -8 + \left(\frac{1}{4}\right)^2$[Add $\left(\frac{1}{4}\right)^2$ to both the sides]

$\left(2x - \frac{1}{4}\right)^2 = -8 + \frac{1}{16} \Rightarrow \left(2x - \frac{1}{4}\right)^2 = \frac{-128+1}{16}$

$\Rightarrow \left(2x - \frac{1}{4}\right)^2 = \frac{-127}{16} < 0$ There are no roots.The roots are imaginary

$2ab = 2x$

$2(2x)b = \mathbf{2}x$

$b = \frac{2x}{4x} = \frac{1}{2}$

$b^2 = \left(\frac{1}{2}\right)^2$

2. Find the roots of the quadratic equations given in Q.1 above by applying the quadratic formula

(i) $\mathbf{2x^2 - 7x + 3 = 0}$ this is in the form of $ax^2 + bx + c = 0$

$a = 2$, $b = -7$, $c = 3$

Roots are, $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x = \frac{-(-7)\pm\sqrt{(-7)^2-4(2)(3)}}{2(2)} \Rightarrow x = \frac{7\pm\sqrt{49-24}}{4}$

$\Rightarrow x = \frac{7\pm\sqrt{25}}{4} \Rightarrow x = \frac{7\pm5}{4}$

$\Rightarrow x = \frac{7+5}{4}, \quad x = \frac{7-5}{4} \quad \Rightarrow x = \frac{12}{4}, \quad x = \frac{2}{4} \quad \Rightarrow x = 3, \quad x = \frac{1}{2}$

$2x^2 + x - 4 = 0$ this is in the form of $ax^2 + bx + c = 0$

(ii) $a = 2, \quad b = 1, \quad c = -4$

Roots are, $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x = \frac{-(1)\pm\sqrt{(1)^2-4(2)(-4)}}{2(2)} \Rightarrow x = \frac{-1\pm\sqrt{1+32}}{2} \Rightarrow x = \frac{-1\pm\sqrt{33}}{2}$

$x = \frac{-1+\sqrt{33}}{2}, \quad x = \frac{-1-\sqrt{33}}{2}$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$ this is in the form of $ax^2 + bx + c = 0$

$a = 4, \quad b = 4\sqrt{3}, \quad c = +3$

Roots are, $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x = \frac{-(4\sqrt{3})\pm\sqrt{(4\sqrt{3})^2-4(4)(3)}}{2(4)}$

$x = \frac{-4\sqrt{3}\pm\sqrt{16\times3-48}}{8} \Rightarrow x = \frac{-4\sqrt{3}\pm\sqrt{48-48}}{8}$

$\Rightarrow x = \frac{-4\sqrt{3}\pm0}{8}, \Rightarrow x = \frac{-4\sqrt{3}}{8}, x = \frac{-4\sqrt{3}}{8} \quad \Rightarrow x = \frac{-\sqrt{3}}{2}, \quad x = \frac{-\sqrt{3}}{2}$

(iv) $2x^2 + x + 4 = 0$ this is in the form of $ax^2 + bx + c = 0$

$a = 2, \quad b = 1, \quad c = 4$

Roots are, $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x = \frac{-(1)\pm\sqrt{(1)^2-4(2)(4)}}{2(2)} \Rightarrow x = \frac{-1\pm\sqrt{1-32}}{4} \Rightarrow x = \frac{-1\pm\sqrt{-31}}{4}$

There is no real root for this equation.

3. Find the roots of the following equations:

(i) $x + \frac{1}{x} = 3, \quad x \neq 0$ (ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, \quad x \neq -4, \quad 7$

(i) $x - \frac{1}{x} = 3, \quad x \neq 0$

$x - \frac{1}{x} = 3$ Multiply the equation by x

$x^2 - 1 = 3x \Rightarrow x^2 - 3x - 1 = 0$ this is in the form of $ax^2 + bx + c = 0$

$a = 1, \quad b = -3, \quad c = -1$

Roots are, $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x = \frac{-(-3)\pm\sqrt{(-3)^2 - 4(1)(-1)}}{2(1)}$

$x = \frac{3\pm\sqrt{9+4}}{2} \Rightarrow x = \frac{3\pm\sqrt{13}}{2} \Rightarrow x = \frac{3+\sqrt{13}}{2}, \quad x = \frac{3-\sqrt{13}}{2}$

(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, \quad x \neq -4, \quad 7$

$30(x-7) - 30(x+4) = 11(x+4)(x-7)$

$30x - 210 - 30x - 120 = 11(x^2 + 4x - 7x - 28)$

$-330 = 11x^2 - 11(3x) - 11(28) \Rightarrow -330 = 11x^2 - 33x - 308$

$\Rightarrow -330 = 11x^2 - 33x - 308 \Rightarrow 11x^2 - 33x + 22 = 0$

$\Rightarrow x^2 - 3x + 2 = 0$ this is in the form of $ax^2 + bx + c = 0$

$a = 1, \quad b = -3, \quad c = 2$

Roots are, $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x = \frac{-(-3)\pm\sqrt{(-3)^2-4(1)(2)}}{2(1)} \Rightarrow x = \frac{3\pm\sqrt{9-8}}{2}$

$\Rightarrow x = \frac{3\pm\sqrt{1}}{2} \Rightarrow x = \frac{3\pm1}{2} \Rightarrow x = \frac{3+1}{2},\ x = \frac{3-1}{2} \Rightarrow x = \frac{4}{2},\ x = \frac{2}{2}$

4. The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now it is $\frac{1}{3}$ Find the present age.

Let the present age of Rehman $= x$ Years

Age of Rehman before 3 years $= (x-3)$ years

His age after five years from now $= (x+5)$ Years.

Sum of the reciprocals of the age $= \frac{1}{3}$

$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$, .

$3(x+5) + 3(x-3) = (x-3)(x+5)$

$3x + 15 + 3x - 9 = x^2 + 2x - 15 \Rightarrow 6x + 15 - 9 = x^2 + 2x - 15$

$\Rightarrow x^2 + 2x - 15 = 6x + 6 \Rightarrow x^2 - 4x - 21 = 0$

$\Rightarrow x^2 - 7x + 3x - 21 = 0 \Rightarrow x(x-7) + 3(x-7) = 0$

$\Rightarrow (x-7)(x+3) = 0 \Rightarrow x - 7 = 0,\ x + 3 = 0$

$\Rightarrow x = 7,\quad x = -3$

$\Rightarrow$The present age of Rehman $= 7$ Years.

5. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Let the marks in Mathematics $= x$ then the marks in English $= 30 - x$ By question,

$(x+2)(30-x-3) = 210 \Rightarrow (x+2)(27-x) = 210$

$\Rightarrow 27x - x^2 + 54 - 2x - 210 = 10$

$\Rightarrow -x^2 + 25x - 156 = 0 \qquad\qquad \times\ -1$

$\Rightarrow x^2 - 25x + 156 = 0 \Rightarrow x^2 - 12x - 13x - 156 = 0$

$\Rightarrow x(x-12) - 13(x-12) = 0 \Rightarrow (x-12)(x-13) = 0$

$\Rightarrow x - 12 = 0,\ x - 13 = 0 \quad \Rightarrow x = 12,\quad x = 13$

If the marks in Mathematics $= 12$ then the marks in English $= 30 - 12 = 18$

If the marks in Mathematics $= 13$ hen the marks in English$= 30 - \mathbf{13} = \mathbf{17}$

6. The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Let the length of the shorter side $= x$ m; Length of the longer side $= (x+30)$m

Length of the diagonals $= (x+60)$m

By Pythagoras theorem length of the diagonals $= \sqrt{x^2 + (x+30)^2}$

$\sqrt{x^2 + (x+30)^2} = x + 60 \Rightarrow x^2 + (x+30)^2 = (x+60)^2$

$\Rightarrow x^2 + x^2 + 2(x)(30) + (30)^2 = x^2 + 2(x)(60) + (60)^2$

$\Rightarrow 2x^2 + 60x + 900 = x^2 + 120x + 3600$

$\Rightarrow 2x^2 - x^2 + 60x - 120x + 900 - 3600 = 0$

$\Rightarrow x^2 - 60x - 2700 = 0 \Rightarrow x^2 - 90x + 30x - 2700 = 0$

$\Rightarrow x(x-90) + 30(x-90) = 0 \Rightarrow (x-90)(x+30) = 0$

$\Rightarrow (x-90) = 0,\ (x+30) = 0 \quad \Rightarrow x = 90,\ x = -30$

$\Rightarrow$ಆಯತದ ದೊಡ್ಡ ಬಾಹುವಿನ ಉದ್ದ $= (x+30) = 90 + 30 = 120$ m

7. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Let the larger and smaller numbers be x and respectively. According to question,

$x^2 - y^2 = 180$ ಮತ್ತು $y^2 = 8x \Rightarrow x^2 - 8x - 180 = 0$

$\Rightarrow x^2 - 18x + 10x - 180 = 0 \Rightarrow x(x-18) + 10(x-18) = 0$

$\Rightarrow (x-18)(x+10) = 0 \Rightarrow x - 18 = 0, \quad x + 10 = 0$

$\Rightarrow x = 18, \quad x = -10$

$\Rightarrow$Larger number $x = 18$

$\therefore\ y^2 = 8x = 8 \times 18 = 144 \Rightarrow y = \pm\sqrt{144} = \pm 12$

$\therefore$ smaller number $= \pm 12$

The numbers are 18 and 12

8. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Speed of the train $= x$ km/h

The time taken to travel 360 km $= \frac{360}{x}$ h

If speed is 5 km/h more, then the required time is $= \frac{360}{x+5}$ h

$\Rightarrow \frac{360}{x} = \frac{360}{x+5} + 1 \Rightarrow 360(x+5) = 360x + x(x+5)$

$\Rightarrow 360x+1800 = 360x + x^2 + 5x \Rightarrow 360x+1800 = 360x + x^2 + 5x$

$\Rightarrow x^2 + 5x - 1800 = 0 \Rightarrow x^2 + 45x - 40x - 1800 = 0$

$\Rightarrow x(x+45) - 40(x+45) = 0 \Rightarrow (x+45)(x-40) = 0$

$\Rightarrow x + 45 = 0, \quad x - 40 = 0 \Rightarrow x = -45, \ x = 40$

Speed of the train $= 40$ km/h

9. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

The time taken by the smaller tap to fill the tank $=$ x hour

The time taken by the larger tap to fill the tank $= (x-10)$ hour

The part of the tank filled by the smaller tap in one hourThe smaller tap $= \frac{1}{x}$

The part of the tank filled by the smaller tap in one hourThe larger tap $= \frac{1}{x-10}$

The time taken by both the tap to fill the tank $= 9\frac{3}{8} = \frac{75}{8}$

The part of the tank filled by both the tap in one hourThe smaller tap $= \frac{1}{\frac{75}{8}} = \frac{8}{75}$

$\frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$

$\Rightarrow 75(x-10) + 75x = 8(x)(x-10) \Rightarrow 75x - 750 + 75x = 8x^2 - 80x$

$\Rightarrow 8x^2 - 80x - 150x + 750 = 0 \Rightarrow 8x^2 - 230x + 750 = 0$

$\Rightarrow 8x^2 - 200x - 30x + 750 = 0 \Rightarrow 8x(x-25) - 30(x-25) = 0$

$\Rightarrow (x-25)(8x-30) = 0 \Rightarrow x - 25 = 0, \quad 8x - 30 = 0$

$\Rightarrow x = 25, \ x = \frac{30}{8} = \frac{15}{4} = 3.75$

If the time taken by the smaller tap to fill the tank $= 3.75$ hr, then The time taken by the larger tap to fill the tank can not be negative.Therefore The time taken by the smaller tap to fill the tank = 25 hour. The time taken by the larger tap to fill the tank 25-10=15 Hours

10. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11km/h more than that of the passenger train, find the average speed of the two trains.

Let the average speed of the passenger train $= x$ km/h;

The average speed of express train $= (x + 11)$km/h

The total distance to travel $= 132$ km

The time taken by passenger train $= \frac{132}{x}$ h

The time taken by express train $= \frac{132}{x+11}$

When the difference between the time taken by two trains is 1 hour

$\therefore \frac{132}{x} - \frac{132}{x+11} = 1$

$132(x + 11) - 132x = x(x + 11) \Rightarrow 132x + 1452 - 132x = x^2 + 11x$

$\Rightarrow x^2 + 11x - 1452 = 0 \Rightarrow x^2 + 44x - 33x - 1452 = 0$

$\Rightarrow x(x + 44) - 33(x + 44) = 0 \Rightarrow (x + 44)(x - 33) = 0$

$\Rightarrow x + 44 = 0,\ x - 33 = 0 \Rightarrow x = -44,\ \ x = 33$

The speed can not be negative

Therefore the average speed of passenger train $= 33$ km/h

The average speed of the express train $= (33 + 11) = 44$ km/h

11. Sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the sides of the two squares.

Let the side of first square be 'x' m and the side of the second square be y

According to question, $4x - 4y = 24m \Rightarrow 4x = 4y + 24 \Rightarrow x = (y + 6)m$

And $x^2 + y^2 = 468 \Rightarrow (y + 6)^2 + y^2 = 468 \Rightarrow y^2 + 12y + 36 + y^2 = 468$

$\Rightarrow 2y^2 + 12y - 432 = 0 \Rightarrow y^2 + 6y - 216 = 0$

$\Rightarrow y^2 + 18y - 12y - 216 = 0$

$\Rightarrow y(y + 18) - 12(y + 18) = 0$

$\Rightarrow (y - 12)(y + 18) = 0$

$\Rightarrow y = 12$ and $y = -18$ [$y = -18$ is not possible]

Therefore the side of the second square is 12m and

the side of the first square is $(12 + 6) = 18$ m

10.5 Nature of Roots

The value of $b^2 - 4ac$ decides the roots of quadratic equation $ax^2 + bx + c = 0$ has real or not, therefore

$b^2 - 4ac$ is called the discriminant of this quadratic equation.and denoted by Δ [delta]

So, the quadratic equation $ax^2 + bx + c = 0$ has

Discriminant	Nature of the roots
$\Delta = 0$	Two equal real roots
$\Delta > 0$	Two distinct real roots
$\Delta < 0$	No real roots

Example 16 : Find the discriminant of the quadratic equation $2x^2 - 4x + 3 = 0$, and hence find the nature of its roots

$a = 2,\quad b = -4,\quad c = 3$

$\Delta = b^2 - 4ac \Rightarrow \Delta = (-4)^2 - 4(2)(3)$

$\Rightarrow \Delta = 16 - 24 \Rightarrow \Delta = -8 < 0$ Roots are imaginary

Example 17 : A pole has to be erected at a point on the boundary of a circular park of diameter 13 metres in such a way that the differences of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Is it possible to do so? If yes, at what distances from the two gates should the pole be erected?

BP = x m ;AP = (x + 7)m ; Diameter AB = 13 m

$\angle APB = 90^0 \Rightarrow AP^2 + PB^2 = AB^2$

$\Rightarrow x^2 + (x + 7)^2 = 13^2 \Rightarrow x^2 + x^2 + 7^2 + 2(x)(7) = 169$

$\Rightarrow 2x^2 + 14x + 49 - 169 = 0 \Rightarrow 2x^2 + 14x - 120 = 0 \quad \div 2$

$\Rightarrow x^2 + 7x - 60 = 0 \Rightarrow x^2 + 12x - 5x - 60 = 0$

$\Rightarrow x(x + 12) - 5(x + 12) = 0 \Rightarrow (x + 12)(x - 5) = 0$

$\Rightarrow x + 12 = 0, \quad x - 5 = 0$

$\Rightarrow x = -12, \quad x = 5$

$\Rightarrow$ x= -12 is not possible. Therefore BP = x m = 5m

AP = (x + 7) = 5 + 7 = 12 m

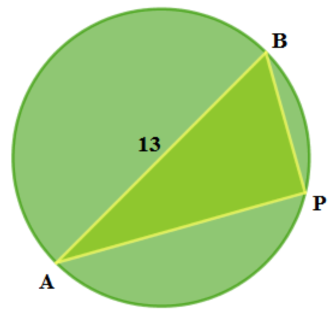


Fig 10.4

Find the discriminant of the equation $3x^2 - 2x + \frac{1}{3} = 0$ and hence find the nature of the roots. Find them if they are real.

$a = 3, \quad b = -2, \quad c = \frac{1}{3}$

$b^2 - 4ac = (-2)^2 - 4(3)\left(\frac{1}{3}\right) = 4 - 4 = 0$

$b^2 - 4ac = 0$ Roots are real and equal

Roots are: $\frac{-b}{2a}, \quad \frac{-b}{2a} = \frac{-(-2)}{2(3)}, \quad \frac{-(-2)}{2(3)} = \frac{2}{6}, \quad \frac{2}{6} = \frac{1}{3}, \quad \frac{1}{3}$

Exercise: 10.4

1. **Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:.(i) $2x^2 - 3x + 5 = 0$ (ii) $3x^2 - 4\sqrt{3}x + 4 = 0$ (iii) $2x^2 - 6x + 3 = 0$**

(i) $2x^2 - 3x + 5 = 0$

$a = 2, \quad b = -3, \quad c = 5$

$\Delta = b^2 - 4ac$

$\Delta = (-3)^2 - 4(2)(5) \Rightarrow \Delta = 9 - 40$

$\Rightarrow \Delta = -31 \Rightarrow \Delta < 0 \Rightarrow$ Roots are imaginary

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

$a = 3, \quad b = -4\sqrt{3}, \quad c = 4$

$\Delta = b^2 - 4ac$

$\Delta = (-4\sqrt{3})^2 - 4(3)(4) \Rightarrow \Delta = 48 - 48$

$\Delta = 0 \Rightarrow$ Roots are real and equal

The roots are: $\frac{-b}{2a}, \quad \frac{-b}{2a} = \frac{-(-4\sqrt{3})}{2(3)}, \quad \frac{-(-4\sqrt{3})}{2(3)} = \frac{4\sqrt{3}}{6}, \quad \frac{4\sqrt{3}}{6}$

$= \frac{2\sqrt{3}}{3}, \quad \frac{2\sqrt{3}}{3} \Rightarrow \frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

(iii) $2x^2 - 6x + 3 = 0$

$a = 2, \quad b = -6, \quad c = 3$

$\Delta = b^2 - 4ac$

$\Delta = (-6)^2 - 4(2)(3) \Rightarrow \Delta = 36 - 24$

$\Rightarrow \Delta = 12 \Rightarrow \Delta > 0 \Rightarrow$ Roots are real and distinct

The roots $= \frac{-b+\sqrt{\Delta}}{2a}, \frac{-b-\sqrt{\Delta}}{2a}$

$= \frac{-(-6)+\sqrt{12}}{2(2)}, \frac{-(-6)-\sqrt{12}}{2(2)} = \frac{6+\sqrt{12}}{4}, \frac{6-\sqrt{12}}{4}$

$= \frac{6+2\sqrt{3}}{4}, \frac{6-2\sqrt{3}}{4} = \frac{3+\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2}$

2. **Find the values of k for each of the following quadratic equations, so that they have two equal roots (i) $2x^2 + kx + 3 = 0$ (ii) $kx(k - 2) + 6 = 0$**

(i) $2x^2 + kx + 3 = 0$ $a = 2,\ b = k,\quad c = 3$

$b^2 - 4ac = 0$

$(k)^2 - 4(2)(3) = 0 \Rightarrow k^2 - 24 = 0 \Rightarrow k^2 = 24$

$k = \pm\sqrt{24} = \pm\sqrt{4 \times 6} = \pm 2\sqrt{6}$

(ii) $kx(x-2) + 6 = 0$

$kx^2 - 2kx + 6 = 0 \Rightarrow a = k,\quad b = -2k,\quad c = 6$

$b^2 - 4ac = 0$

$\Rightarrow (-2k)^2 - 4(k)(6) = 0 \Rightarrow 4k^2 - 24k = 0$

$\Rightarrow 4k(k-6) = 0 \Rightarrow 4k = 0, k - 6 = 0$

$\Rightarrow k = 0,\quad k = 6$

3. **Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2? If so, find its length and breadth**

The breadth of the mango grove $= l$;The length $= 2l$

The area of the grove = Length x breadth

$\Rightarrow (l)(2l) = 800 \Rightarrow 2l^2 = 800 \quad \Rightarrow l^2 = \frac{800}{2} = 400 \Rightarrow l = \pm\sqrt{400} = \pm 20$

∴ The breadth of the mango grove $= l = 20$ m

∴ The breadth of the mango grove $= 2l = 2 \times 20 = 40$ m

4. **Is the following situation possible? If so, determine their present ages.The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.**

Let the age of A friend $= x$ Years

The age of B friend $= (20 - x)$ years

The age of friend A before 4 $= (x - 4)$

The age of B friend before 4 years $= (20 - x - 4) = 16 - x$

$(x-4)(16-x) = 48$

$16x - x^2 - 64 + 4x = 48$

$-x^2 + 20x - 64 - 48 = 0$

$x^2 - 20x + 112 = 0$

$a = 1,\quad b = -20,\quad c = 112$

$b^2 - 4ac = (-20)^2 - 4(1)(112)$

$= 400 - 448 = -48$

The equation has no real roots. Therefore this situation is not possible

5. **Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2? If so, find its length and breadth.**

Let the length and breadth of the rectangle be l and b; The perimeter $= 2(l + b) = 80$

$l + b = \frac{80}{2} = 40 \Rightarrow l = 40 - b$

Area $l \times b = 400 \Rightarrow l(40 - l) = 400$

$\Rightarrow 40l - l^2 = 400 \Rightarrow l^2 - 40l + 400 = 0$

$a = 1,\quad b = -40,\quad c = 400$

$b^2 - 4ac = (-40)^2 - 4(1)(400) = 1600 - 1600 = 0$

$b^2 - 4ac = 0$ Roots are rea and equal

Roots are: $\frac{-b}{2a}, \quad \frac{-b}{2a} = \frac{-(-40)}{2(1)}, \quad \frac{-(-40)}{2(1)} = \frac{40}{2}, \quad \frac{40}{26} = 20, \quad 20$

Length $= 20\ m$; Breadth $b = 40 - l = 40 - 20 = 20m$

Summary:

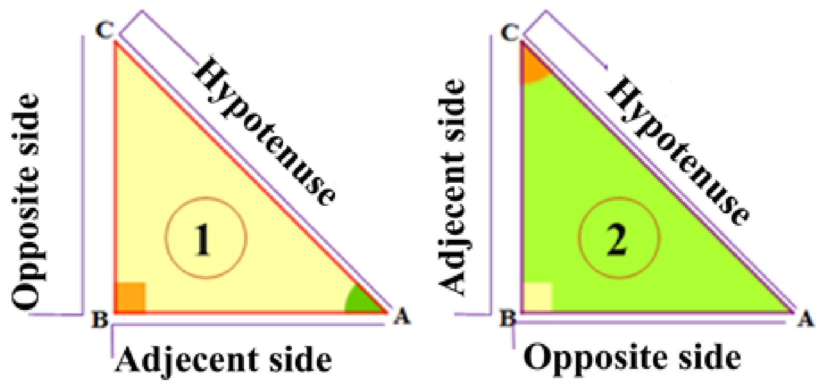
1. A quadratic equation in the variable x is of the form $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$
2. A real number is said to be a root of the quadratic equation $ax^2 + bx + c = 0$, if $a^2 + b + c = 0$. The zeroes of the quadratic polynomial $ax^2 + bx + c$ and the roots of the quadratic equation $ax^2 + bx + c = 0$ are the same.
3. If we can factorise $ax^2 + bx + c$, a 0, into a product of two linear factors, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.
4. A quadratic equation can also be solved by the method of completing the square.
5. Roots of the quadratic equation $ax^2 + bx + c = 0$ are given by the formula
   $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where b2 - 4ac ≥ 0
6. For the quadratic equation $ax^2 + bx + c = 0$,
   (i) If $b^2 - 4ac > 0$ then roots are real and distinct
   (ii) If $b^2 - 4ac = 0$ roots are real and equal
   (iii) If $b^2 - 4ac < 0$ no real roots

11 ITRODUCTION TO TRIGONOMETRY

Trigonometry is the study of relationships between the sides and angles of a triangle.

11.2 Trigonometric Ratios:

To know the trigonometric ratio we have to consider right angle triangle.



There are six trigonometric ratios:

Trigonometric ratios		Triangle 1	Triangle 2
SinA	$\frac{\text{Opposite}}{\text{Hypotenuse}}$	$\frac{BC}{AC}$	$\frac{AB}{AC}$
CosA	$\frac{\text{Adjecent}}{\text{Hypotenuse}}$	$\frac{AB}{AC}$	$\frac{BC}{AB}$
Tan A	$\frac{\text{Opposite}}{\text{Adjecent}}$	$\frac{BC}{AB}$	$\frac{AB}{BC}$
CosecA	$\frac{\text{Hypotenuse}}{\text{Opposite}}$	$\frac{AC}{BC}$	$\frac{AC}{AB}$
SecA	$\frac{\text{Hypotenuse}}{\text{Adjecent}}$	$\frac{AC}{AB}$	$\frac{AC}{BC}$
CotA	$\frac{\text{Adjecent}}{\text{Opposite}}$	$\frac{AB}{BC}$	$\frac{BC}{AB}$

Example 1: tan A = $\frac{4}{3}$ find the other trigonometric ratios of the angle A

In ΔABC, $\angle ABC = 90^0$ $\therefore$ By Pythagoras theorem,

$AC^2 = AB^2 + BC^2 \Rightarrow AC^2 = 4^2 + 3^3 = 16 + 9 = 25 \Rightarrow AC = 5$

$\text{SinA} = \frac{BC}{AC} = \frac{4}{5}$; $\text{Cos A} = \frac{AB}{AC} = \frac{3}{5}$; $\text{TanA} = \frac{BC}{AB} = \frac{4}{3}$

$\text{CosecA} = \frac{AC}{BC} = \frac{5}{4}$; $\text{SecA} = \frac{AC}{AB} = \frac{5}{3}$; $\text{CotA} = \frac{AB}{BC} = \frac{3}{4}$

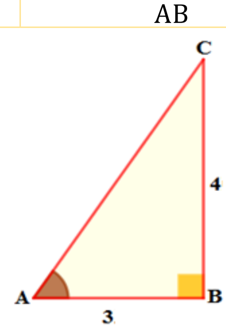
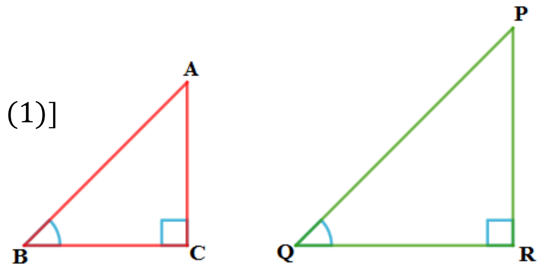


Fig 11.8

Example 2 :If B and Q are acute angles such that sin B = sin Q, then prove that ∠B = ∠Q.

Sin B = SinQ $\Rightarrow \frac{AC}{AB} = \frac{PR}{PQ}$

$\Rightarrow \frac{AC}{PR} = \frac{AB}{PQ} = k$ (1)



BC = $\sqrt{AB^2 - AC^2}$ [By Pythagoras theorem]

$\Rightarrow \sqrt{k^2PQ^2 - k^2PR^2} \Rightarrow k.\sqrt{PQ^2 - PR^2}$ [from (1)]

QR = $\sqrt{PQ^2 - PR^2}$

$\Rightarrow \frac{BC}{QR} = \frac{k.\sqrt{PQ^2-PR^2}}{\sqrt{PQ^2-PR^2}} = k$ (2)

From (1) and (2),

$\frac{AC}{PR} = \frac{AB}{PQ} = \frac{BC}{QR} \Rightarrow \Delta ABC \sim \Delta PQR$

∴ **∠B = ∠Q**

Example 3 : Consider ΔACB , right-angled at C, in which AB = 29 units, BC = 21 units and ∠ABC = θ (see Fig. 11.10). Determine the values of

$Cos^2\theta + Sin^2\theta$ (ii) $Cos^2\theta$ - $Sin^2\theta$

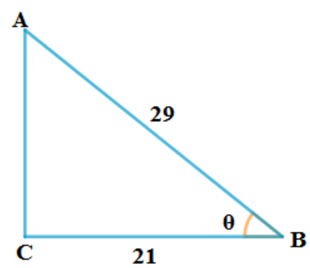


Fig 11.10

In right angle triangle ACB, ∠ACB = 90^0

Therefore AC = $\sqrt{AB^2 - BC^2} \Rightarrow AC = \sqrt{29^2 - 21^2}$

$\Rightarrow AC = \sqrt{841 - 441} = \sqrt{400} = 20$

(i) $Cos^2\theta + Sin^2\theta$

$= \frac{21^2}{29^2} + \frac{20^2}{29^2} = \frac{441+400}{841} = \frac{841}{841} = 1$

(ii) $Cos^2\theta$ - $Sin^2\theta$

$= \frac{21^2}{29^2} - \frac{20^2}{29^2} = \frac{441-400}{841} = \frac{41}{841} = 1$

Example 4 : In a right triangle ABC, right-angled at B, if tan A = 1, then verify that 2sin A cos A = 1

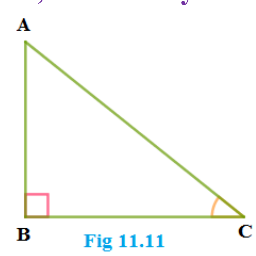


Fig 11.11

In right angle triangle ACB ,

tan A = 1 $\Rightarrow \frac{AB}{BC} = 1 \Rightarrow AB = BC$

$AC^2 = AB^2 + BC^2$ [By Pythagoras theorem]

$\Rightarrow AC^2 = 2AB^2$ (1)

Now, 2sin A cos A $= 2.\frac{AB}{AC}.\frac{BC}{AC} = 2.\frac{AB^2}{AC^2} = 2.\frac{AB^2}{2AB^2} = 1$

Example 5 : In ΔOPQ right-angled at P, OP = 7 cm and OQ – PQ = 1 cm (see Fig. 11.12). Determine the values of sin Q and cos Q.

Fig 11.12

In ΔOPQ,

$OQ^2 = PQ^2 + OP^2$ [By Pythagoras theorem]

$\Rightarrow (1 + PQ)^2 = PQ^2 + 7^2 \Rightarrow 1 + PQ^2 + 2PQ = PQ^2 + 49$

$\Rightarrow 1 + 2PQ = 49$

$\Rightarrow 2PQ = 49 - 1 = 48 \Rightarrow PQ = 24cm$

$\Rightarrow$ OQ = 1 + PQ= 1 + 24 $\Rightarrow$ OQ = 25

∴ **sin Q = $\frac{7}{25}$ and cos Q = $\frac{24}{25}$**

Inverse of trigonometric values		
$\frac{1}{\text{SinA}}$	$\frac{\text{Hypotenuse}}{\text{Opposite}}$	**CosecA**
$\frac{1}{\text{CosA}}$	$\frac{\text{Hypotenuse}}{\text{Adjecent}}$	**SecA**
$\frac{1}{\text{Tan A}}$	$\frac{\text{Adjecent}}{\text{Opposite}}$	**CotA**
$\frac{1}{\text{CosecA}}$	$\frac{\text{Opposite}}{\text{Hypotenuse}}$	**SinA**
$\frac{1}{\text{SecA}}$	$\frac{\text{Adjecent}}{\text{Hypotenuse}}$	**SecA**
$\frac{1}{\text{CotA}}$	$\frac{\text{Opposite}}{\text{Adjecent}}$	**CotA**

Exercise 11.1

[for solving problems, the value of constant k is taken as 1]

1. In ΔABC, right-angled at B, AB = 24 cm, BC = 7 cm. Determine
i) sin A, cos A (ii) sin C, cos C

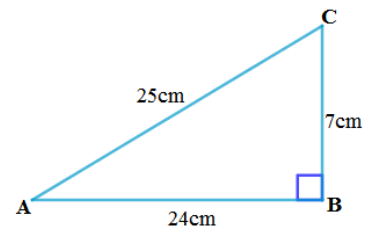
In Δ ABC, $\angle B = 90^o$ $\therefore$ by Pythagoras theorem,

$AC^2 = AB^2 + BC^2 = (24)^2 + 7^2$

$= (576+49)\ cm^2 = 625\ cm^2 \Rightarrow AC = 25$

(i) $\sin A = \frac{BC}{AC} = \frac{7}{25}$, $\cos A = \frac{AB}{AC} = \frac{24}{25}$

(ii) $\sin C = \frac{AB}{AC} = \frac{24}{25}$; $\cos C = \frac{BC}{AC} = \frac{7}{25}$



2. In Fig. 8.13, find tan P – cot R.

In ΔPQR, By Pythagoras theorem,

$QR^2 = PR^2 - PQ^2 = (13)^2 - (12)^2 = 169 - 144$

$\Rightarrow QR^2 = 25 \Rightarrow QR = 5$ cm

ಈಗ, $\tan P = \frac{QR}{PQ} = \frac{5}{12}$

$\cot R = \frac{QR}{PQ} = \frac{5}{12}$

$\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$

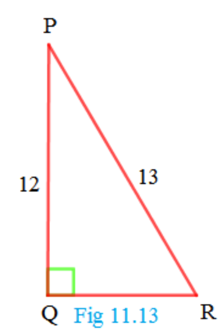


Fig 11.13

3. If $\sin A = \frac{3}{4}$ calculate the value of cos A and tan A

In ΔABC, $\angle B = 90^0$ According to question,

$\sin A = \frac{3}{4} = \frac{BC}{AC} \Rightarrow AC = 4k, BC = 3k$ [Here, Let k = 1]

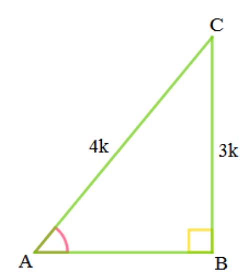
$\therefore$ By Pythagoras theorem,

$AB^2 = AC^2 - BC^2 \Rightarrow AB^2 = AC^2 - BC^2$

$AB^2 = 4^2 - 3^2 = 16 - 9 = 7 \Rightarrow AB = \sqrt{7}$

$\therefore \cos A = \frac{AB}{AC} = \frac{\sqrt{7}}{4}$

$\tan A = \frac{Bc}{AB} = \frac{3}{\sqrt{7}}$



4. **Given 15 cot A = 8 find sin A and sec A**

According to question, cot A = $\frac{8}{15} = \frac{BC}{AB}$

In ΔABC, $\angle B = 90^0$

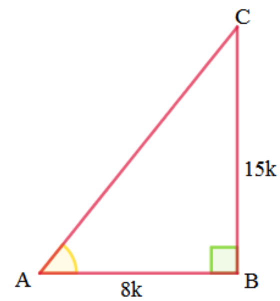
BC = 15k, AB = 8k [Here, Let k = 1]

∴ By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$AC^2 = 8^2 + 15^2 = 64 + 225 = 289 \Rightarrow AC = 17$

$\Rightarrow \sin A = \frac{BC}{AC} = \frac{15}{17}$; $\sec A = \frac{AC}{AB} = \frac{17}{8}$



5. **Given $\sec\theta = \frac{13}{12}$ calculate all other trigonometric values.**

According to question, $\sec\theta = \frac{13}{12} = \frac{OP}{OM}$

In ΔPMO, $\angle M = 90^0$

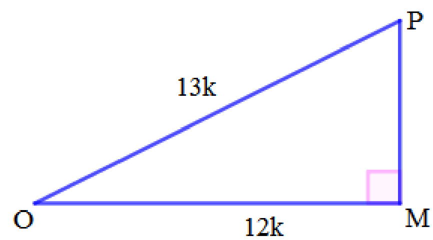
OM = 12k, OP = 13k [Here, Let k = 1]

∴ By Pythagoras theorem, $PM^2 = OP^2 + OM^2$

$PM^2 = 13^2 + 12^2 = 169 - 144 = 25 \Rightarrow AC = 5$

$\sin\theta = \frac{MP}{OP} = \frac{5}{13}$; $\cos\theta = \frac{OM}{OP} = \frac{12}{13}$; $\tan\theta = \frac{MP}{OM} = \frac{5}{12}$

$\cot\theta = \frac{OM}{MP} = \frac{12}{5}$; $\operatorname{cosec}\theta = \frac{OP}{MP} = \frac{13}{5}$



6. **If A and B are acute angles such that cos A = cos B, then show that ∠A = ∠B**

In ΔABC, $CD \perp AB$.

According to question, cos A = cos B

$\Rightarrow \frac{AD}{AC} = \frac{BD}{BC} \Rightarrow \frac{AD}{BD} = \frac{AC}{BC}$

$\frac{AD}{BD} = \frac{AC}{BC} = k$ ಆಗಿರಲಿ.

$\Rightarrow AD = kBD$ (1)

$\Rightarrow AC = kBC$ (2)

In ΔCAD and ΔCBD, By Pythagoras theorem,

$CD^2 = AC^2 - AD^2$ (3)

$CD^2 = BC^2 - BD^2$ (4)

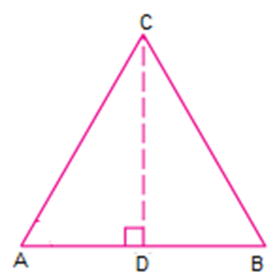
From (3) and (4), $AC^2 - AD^2 = BC^2 - BD^2$

$\Rightarrow (kBC)^2 - (k\,BD)^2 = BC^2 - BD^2 \Rightarrow k^2(BC^2 - BD^2) = BC^2 - BD^2$

$\Rightarrow k^2 = 1 \Rightarrow k = 1$

Substitute k = 1 in (2),

AC = BC $\Rightarrow \angle A = \angle B$ [Angle opposite to the equal sides are equal]



7. **If $\cot\theta = \frac{7}{8}$ then find the value of i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ ii) $\cot^2\theta$**

In Δ ABC, $\angle B = 90^o$ and $\angle C = \theta$ According to question,

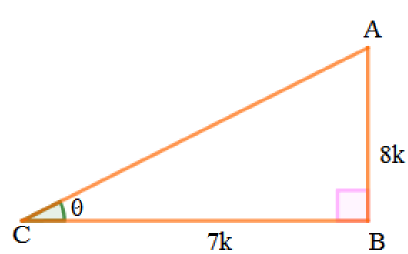
$\cot\theta = \frac{BC}{AB} = \frac{7}{8} \Rightarrow AB = 8$ ಮತ್ತು BC = 7 [If k = 1]

In Δ ABC By Pythagoras theorem,

$AC^2 = AB^2 + BC^2 \Rightarrow AC^2 = 8^2 + 7^2$

$\Rightarrow AC^2 = 64 + 49 \Rightarrow AC^2 = 113 \Rightarrow AC = \sqrt{113}$

$\sin\theta = \frac{AB}{AC} = \frac{8}{\sqrt{113}}$ and $\cos\theta = \frac{BC}{AC} = \frac{7}{\sqrt{113}}$



(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta}$

$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1-\frac{64}{113}}{1-\frac{49}{113}} = \frac{\frac{113-64}{113}}{\frac{113-49}{113}} = \frac{\frac{49}{113}}{\frac{64}{113}} = \frac{49}{64}$

$\cot^2\theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$

8. If 3 cot A = 4 Check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not?

In Δ ABC , ∠B = 90º Given,

$\cot A = \frac{AB}{AC} = \frac{4}{3} \Rightarrow AB = 4$ ಮತ್ತು $BC = 3$, [taken k = 1]

In Δ ABC by Pythagoras theorem,

$AC^2 = AB^2 + BC^2 \Rightarrow AC^2 = 4^2 + 3^2$

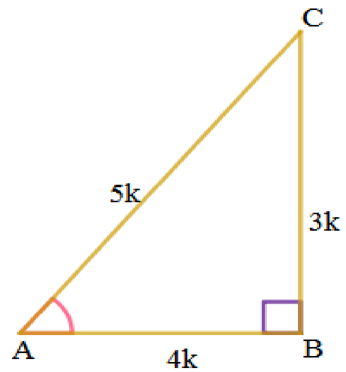
$\Rightarrow AC^2 = 16 + 9 \Rightarrow AC^2 = 25 \Rightarrow AC = 5$

$\tan A = \frac{BC}{AB} = \frac{3}{4}$; $\sin A = \frac{BC}{AC} = \frac{3}{5}$; $\cos A = \frac{AB}{AC} = \frac{4}{5}$

$LHS = \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}} = \frac{\frac{7}{16}}{\frac{25}{16}} = \frac{7}{25}$

$R.H.S. = \cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{4}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

$\Rightarrow R.H.S. = L.H.S. \therefore \frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$



9. In triangle ΔABC, right-angled at B, if $\tan A = \frac{1}{\sqrt{3}}$ then find the value of

i) sin A cos C + cos A sin C ii) cos A cos C - sin A sin C

In Δ ABC , ∠B = 90º Given, $\tan A = \frac{BC}{AB} = \frac{1}{\sqrt{3}}$

Let $AB = \sqrt{3}$ and $BC = 1$

In Δ ABC By Pythagoras theorem,

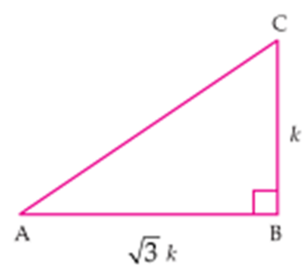
$AC^2 = AB^2 + BC^2 \Rightarrow AC^2 = (\sqrt{3})^2 + (1)^2 \Rightarrow AC^2 = 3 + 1$

$\Rightarrow AC^2 = 4 \Rightarrow AC = 2$

$\sin A = \frac{BC}{AC} = \frac{1}{2}$; $\cos A = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$; $\sin C = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$;$\cos C = \frac{BC}{AC} = \frac{1}{2}$

(i) $\sin A \cos C + \cos A \sin C = \left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}\right) = \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$

(ii) $\cos A \cos C - \sin A \sin C = \left(\frac{\sqrt{3}}{2} \times \frac{1}{2}\right) - \left(\frac{1}{2} \times \frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$



10. In .ΔPQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

Given PR + QR = 25 , PQ = 5

Let $PR = x$. ∴ $QR = 25 - x$

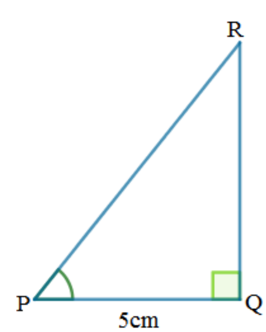
By Pythagoras theorem, $PR^2 = PQ^2 + QR^2$

$x^2 = (5)^2 + (25 - x)^2 \Rightarrow x^2 = 25 + 625 + x^2 - 50x$

$\Rightarrow 50x = 650 \Rightarrow x = 13$

∴ PR = 13 cm ⇒ QR = (25 - 13) cm = 12 cm

$\sin P = \frac{QR}{PR} = \frac{12}{13}$; $\cos P = \frac{PQ}{PR} = \frac{5}{13}$; $\tan P = \frac{QR}{PQ} = \frac{12}{5}$

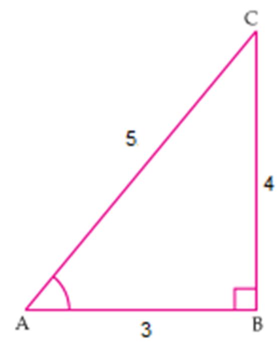


11. State whether the following are true or false. Justify your answer.

i) The value of tan A is always less than 1.

ii) sec A =$\frac{12}{5}$ for some values of A
iii) cos A is the abbreviation used for the cosecant of angle A
iv) cot A is the product of cot and A
v) sin θ =$\frac{4}{3}$ for some values of θ

(i) False
In ΔABC, ∠B = 90^0,
Let AB = 3, BC = 4 and AC = 5 ⇒tan A = $\frac{4}{3}$ > 1



(ii) True
In ΔABC , ∠B = 90^0,
AC =13k and AB = 5k [k Positive real number]
⇒ AC =12,BC=5 and AB = 5 [If k = 1]
$BC^2 = AC^2 - AB^2$ [By Pythagoras theorem]
⇒$BC^2 = 12^2 - 5^2$ ⇒BC^2 = 144 – 25 ⇒BC^2 = 119⇒sec A =$\frac{12}{5}$

(iii) False
cosecA is a abbreviation of CosecantA and CosA is the abbreviation of CosineA.

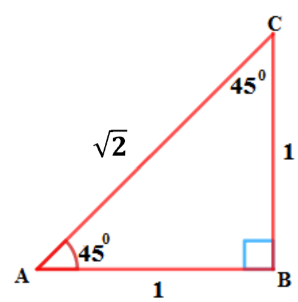
(iv) False
cot A is not a product of cot and A . it is just a symbol

(v) False
sin θ = $\frac{Opposite}{Hypotenuse}$ In any triangle hypotenuse is the larger side.
∴ sin θ is always ≤ 1 ⇒sin θ = $\frac{4}{3}$, it is not possible for any value of θ

11.3 Trigonometric Ratios of Some Specific Angles:

Trigonometric ratio of 45^0

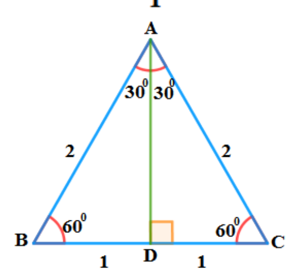
In ΔABC, ∠B = 90^0,
∠A = 45^0 ⇒ ∠C = 45^0 [Sum of interior angles is 180^0]
⇒ Let AB = BC = 1 , By Pythagoras theorem,
∴ $AC^2 = AB^2 + BC^2$ ⇒ $AC^2 = 1^2 + 1^2 = 1+1 = 2$
⇒ AC = $\sqrt{2}$



$Sin45^0$	$\frac{1}{\sqrt{2}}$	$Cosec45^0$	$\sqrt{2}$
$Cos45^0$	$\frac{1}{\sqrt{2}}$	$Sec45^0$	$\sqrt{2}$
$Tan45^0$	1	$Cot45^0$	1

Trigonometric Ratios of 30^0 and 60^0

In equilateral triangle, the angles are equal.
⇒∠A = ∠B = ∠C = 60^0
Draw AD⊥BC ⇒BD = CD
[In an equilateral triangle the perpendicular from the vertex bisects the base]



$\Rightarrow \angle BAD = \angle CAD = 30^0$
Let AB = BC = CA = 2, $\Rightarrow$BD = CD = 1

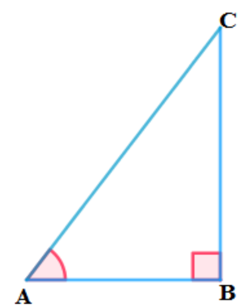
In ΔABD , By Pythagoras theorem,
$AD^2 = AB^2 - BD^2 \Rightarrow AD^2 = 2^2 - 1^2 = 4 - 1 = 3 \Rightarrow AD = \sqrt{3}$

$Sin60^0$	$\frac{\sqrt{3}}{2}$	$Cosec60^0$	$\frac{2}{\sqrt{3}}$
$Cos60^0$	$\frac{1}{2}$	$Sec60^0$	2
$Tan60^0$	$\sqrt{3}$	$Cot60^0$	$\frac{1}{\sqrt{3}}$

$Sin30^0$	$\frac{1}{2}$	$Cosec30^0$	2
$Cos30^0$	$\frac{\sqrt{3}}{2}$	$Sec30^0$	$\frac{2}{\sqrt{3}}$
$Tan30^0$	$\frac{1}{\sqrt{3}}$	$Cot30^0$	$\sqrt{3}$

Trigonometric ratios of 0^0 and 90^0

If $\angle A$ Closer to 0^0 then the length of BC closer to 0 and almost AB = AC



$Sin0^0$	0	$Cosec0^0$	ND
$Cos0^0$	1	$Sec0^0$	1
$Tan0^0$	0	$Cot0^0$	ND

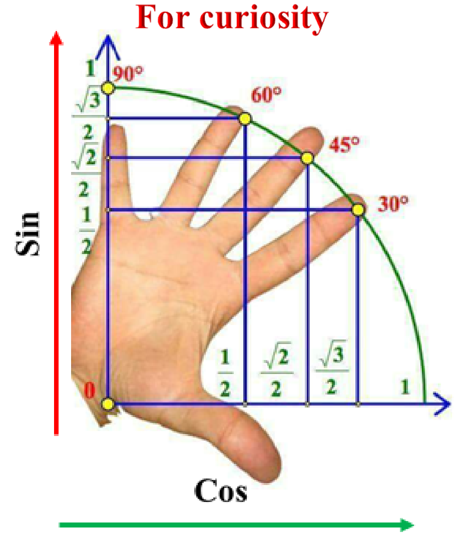
If $\angle A$ closer to 90^0 Then the length of AB closer to 0 and almost AC = AC
Let AB = AC = 1 and BC = 0

$Sin90^0$	0	$Cosec90^0$	ND
$Cos90^0$	1	$Sec90^0$	1
$Tan90^0$	0	$Cot90^0$	ND

Table 11.1

$\angle A$	0^0	30^0	45^0	60^0	90^0
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND
osec	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
Sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND
Cot	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

For curiosity



Example 6 : In ΔABC, right-angled at B, AB = 5 cm and $\angle ACB = 30°$ (see Fig. 11.19). Determine the lengths of the sides BC and AC.

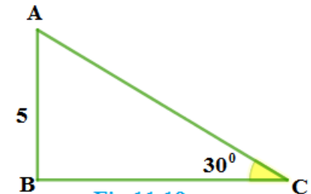


Fig 11.19

$Tan30^0 = \frac{5}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{5}{BC} \Rightarrow 5\sqrt{3}cm$

$Sin30^0 = \frac{5}{AC} \Rightarrow \frac{1}{2} = \frac{5}{AC} \Rightarrow AC = 10cm$

Example 7 : In ΔPQR, right-angled at Q (see Fig. 11.20), PQ = 3 cm and PR = 6 cm. Determine $\angle QPR$ and $\angle PRQ$.

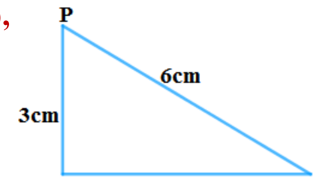


Fig 11.20

$SinR = \frac{3}{6} = \frac{1}{2}$

$\Rightarrow \angle R = 30^0 \Rightarrow \angle PRQ = 30^0$

$\therefore \angle QPR = 60^0$

Example 8: If sin (A - B) = $\frac{1}{2}$ and cos (A + B) = $\frac{1}{2}$, $0 < A + B \le 90$, A > B find A and B

If sin (A - B) = $\frac{1}{2}$ then $\sin 30^0 = \frac{1}{2} \Rightarrow A - B = 30^0$ (1)

If cos (A + B) = $\frac{1}{2}$ then $\cos 60^0 = \frac{1}{2} \Rightarrow A + B = 60^0$ (2)

$(1) + (2) = 2A = 90^0 \Rightarrow A = 45^0$

From (2) $\Rightarrow 45^0 - B = 30^0 \Rightarrow B = 15^0$

Exercise 11.2

1. **Evaluate the following:**

i) $\sin 60^0 \cos 30^0 + \sin 30^0 \cos 60^0$ ii) $2\tan^2 45^0 + \cos^2 30^0 - \sin^2 60^0$

iii) $\frac{\cos 45^0}{\sec 30^0 + cosec\, 30^0}$ iv) $\frac{Sin30^0 + \tan 45^0 - Cosec45^0}{Sec30^0 + \cos 60^0 + \cot 45^0}$ iv) $\frac{5Cos^2 60^0 + 4sec^2 30^0 - tan^2 45^0}{Sin^2 30^0 + \cos^2 30^0}$

i) $\sin 60^0 \cos 30^0 + \sin 30^0 \cos 60^0$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{3}{4} + \frac{1}{4} = 1$

ii) $2\tan^2 45^0 + \cos^2 30^0 - \sin^2 60^0$

$= 2\,(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = 2$

iii) $\frac{\cos 45^0}{Sec\, 30^0 + cosec\, 30^0}$

$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}} = \frac{\sqrt{3}}{\sqrt{2}(2+2\sqrt{3})} = \frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}} = \frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}} \text{ x } \frac{2\sqrt{2}-2\sqrt{6}}{2\sqrt{2}-2\sqrt{6}} = \frac{2\sqrt{6}-2\sqrt{18}}{(2\sqrt{2})^2-(2\sqrt{6})^2}$

$= \frac{2\sqrt{6}-6\sqrt{2}}{4x2 - 4x6} = \frac{2(\sqrt{6}-3\sqrt{2})}{8-24} = \frac{2(\sqrt{6}-3\sqrt{2})}{-16} = \frac{\sqrt{6}-3\sqrt{2}}{-8} = \frac{3\sqrt{2}-\sqrt{6}}{8}$

iv) $\frac{Sin30^0 + \tan 45^0 - Cosec60^0}{Sec30^0 + \cos 60^0 + \cot 45^0}$

$= \frac{\left(\frac{1}{2}\right) + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \frac{\frac{\sqrt{3} + 2\sqrt{3} - 4}{2\sqrt{3}}}{\frac{4 + \sqrt{3} + 2\sqrt{3}}{2\sqrt{3}}} = \frac{\frac{3\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+3\sqrt{3}}{2\sqrt{3}}} = \frac{3\sqrt{3}-4}{4+3\sqrt{3}}$

$= \frac{3\sqrt{3}-4}{4+3\sqrt{3}} \text{ x } \frac{4-3\sqrt{3}}{4-3\sqrt{3}} = \frac{12\sqrt{3} - 16 - 9\sqrt{9} + 12\sqrt{3}}{(4)^2 - (3\sqrt{3})^2}$

$= \frac{12\sqrt{3} - 16 - 27 + 12\sqrt{3}}{16 - 27} = \frac{24\sqrt{3} - 43}{-11} = \frac{43 - 24\sqrt{3}}{11}$

iv) $\frac{5Cos^2 60^0 + 4sec^2 30^0 - tan^2 45^0}{Sin^2 30^0 + \cos^2 30^0}$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - 1}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{1}{4} + \frac{3}{4}} = \frac{\frac{15+64-12}{12}}{1} = \frac{67}{12}$$

2. Choose the correct option and justify your choice:

i) $\frac{2\tan 30^0}{1 + \tan^2 30^0}$

A) $\sin 60^0$ B) $\cos 60^0$ C) $\tan 60^0$ D) $\sin 30^0$

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \frac{\sqrt{3}}{2}$$

Ans: A) $\sin 60^0$

ii) $\frac{1 - \tan^2 45^0}{1 + \tan^2 45^0}$

A) $\tan 90^0$ B) 1 C) $\sin 45^0$ D) 0

$\frac{1-1}{1+1} = \frac{0}{2} = 0$

Ans: D) 0

iii) sin 2A = 2 sin A is true when A =

A) 0 B) 30 C) 45 D) 60

$\sin 2x0 = 2 \sin 0 \Rightarrow \sin 0 = 2 \sin 0 \Rightarrow 0 = 0$

Ans: A) 0

iv) $\frac{2\tan 30^0}{1 - \tan^2 30^0}$

A) $\cos 60^0$ B) $\sin 60^0$ C) $\tan 60^0$ D) $\sin 30^0$

$$\frac{2x\frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

Ans: C) $\tan 60^0$

3. **If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$, $0 < A + B \le 90$; $A > B$ find A and B**

$\tan(A + B) = \sqrt{3} \Rightarrow A + B = 60^0$ (1)

$\tan(A - B) = \frac{1}{\sqrt{3}} \Rightarrow A - B = 30^0$ (2)

(2) – (1) $\Rightarrow 2B = 30^0 \Rightarrow B = 15^0 \Rightarrow$ (1) ರಿಂದ $A = 60 - 15 = 45^0$

4. State whether the following are true or false. Justify your answer.

i) sin (A + B) = sin A + sin B

Let $A = 30^0$ and $B = 90^0$

$\sin(30^0 + 60^0) = \sin 90^0 = 1 \Rightarrow \sin 30^0 + \sin 60^0 = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$

$\therefore \sin(A + B) \neq \sin A + \sin B$

$\therefore$ The statement is false

ii) The value of sin θ increases as θ increases

$\sin 0^0 = 0$, $\sin 90^0 = 1$

$\therefore$ The statement is true

iii)The value of cos θ increases as θ increases.

$\text{Cos } 0^0 = 1$, $\text{Cos } 90^0 = 0$

Here, we observe that as θ increases the value of cos θ dicreases

$\therefore$ The statement is false

iv)sinθ = cosθ for all values of θ

$\sin 30^0 = \frac{1}{2}$; $\cos 30^0 = \frac{\sqrt{3}}{2}$

⇒ $\sin\theta \neq \cos\theta$ for all values of θ

∴ The statement is false

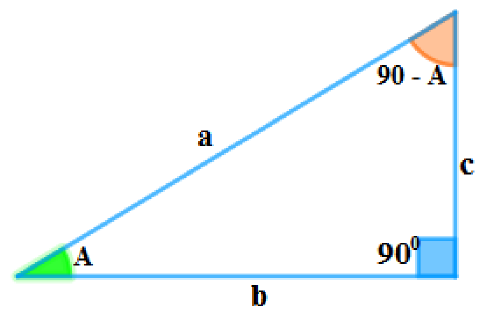
v) cot A is not defined for A = 0^0

The statement is true

11.4 Trigonometric Ratios of Complementary Angles

Two angles are given and if their sum is equal to 90^0 then angles are called complementary Angles

Trigonometric ratios		Trigonometric ratios of complementary angles
SinA	$\frac{c}{a}$	Cos(90-A)
CosA	$\frac{b}{a}$	Sin(90-A)
TanA	$\frac{c}{b}$	Cot(90-A)
CosecA	$\frac{a}{c}$	Sec(90-A)
SecA	$\frac{a}{b}$	Cosec(90-A)
CotA	$\frac{b}{c}$	Tan(90-A)



Example 9 : Evaluate $\frac{\tan 65^0}{\cot 25^0}$

$\frac{\tan 65^0}{\cot 25^0} = \frac{\tan(90-25)^0}{\cot 25^0} = \frac{\cot 25^0}{\cot 25^0} = 1$

Example 10 : If sin 3A = cos (A – 26°), where 3A is an acute angle, find the value

Given $\sin 3A = \cos(A-26)$

$\Rightarrow \cos(90-3A) = \cos(A-26^0) \Rightarrow 90-3A = A-26^0$

$\Rightarrow 90+26 = A+3A \Rightarrow 116 = 4A \Rightarrow A = 29^0$

Example 11 : Express cot 85° + cos 75° in terms of trigonometric ratios of angles between 0^0 and 45^0

$\cot 85^0 = \cot(90-5^0) = \tan 5^0$

$\cos 75^0 = \cos(90-15^0) = \sin 15^0$

Exercise 11.3

1. **Evaluate: i) $\frac{\sin 18^0}{\cos 72^0}$ ii) $\frac{\sin 26^0}{\cos 64^0}$ iii) $\cos 48^0 - \sin 42^0$ vi) $\mathrm{cosec}\,31^0 - \sec 59^0$**

i) $\frac{\sin 18^0}{\cos 72^0}$

$\frac{\sin 18^0}{\cos 72^0} = \frac{\sin(90-72^0)}{\cos 72^0} = \frac{\cos 72^0}{\cos 72^0} = 1$

ii) $\frac{\sin 26^0}{\cos 64^0}$

$\frac{\sin 26^0}{\cos 64^0} = \frac{\sin(90-64^0)}{\cos 64^0} = \frac{\cos 64^0}{\cos 64^0} = 1$

iii) $\cos 48^0 - \sin 42^0$

$\cos 48^0 - \sin(90-48^0) = \cos 48^0 - \cos 48^0 = 0$

vi) $cosec31^0 - sec59^0$

$cosec31^0 - sec59^0 = cosec31^0 - sec(90 - 31^0) = cosec31^0 - cosec31^0 = 0$

2. **Show that i) $\tan48^0 \tan23^0 \tan42^0 \tan67^0 = 1$ ii) $\cos38^0 \cos52^0 - \sin38=^0 \sin52^0 = 0$ i)**

$\tan48^0 \tan23^0 \tan42^0 \tan67^0 = 1$

LHS $= \tan48^0 \tan23^0 \tan(90-48^0) \tan(90-23^0)$

$= \tan48^0 \tan23^0 \cot 48^0 \cot 23^0 = \tan48^0 x \tan23^0 x \frac{1}{\tan48^0} x \frac{1}{\tan23^0} = 1$

ii) $\cos38^0 \cos52^0 - \sin38^0 \sin52^0 = 0$

LHS $= \cos38^0 \cos52^0 - \sin38^0 \sin52^0$

$= \cos38^0 \cos52^0 - \sin(90 - 52^0) \sin(90-38)^0 = \cos38^0 \cos52^0 - \cos52^0 \cos38^0$

$= \cos38^0 \cos52^0 - \cos52^0 \cos38^0 = 0$ RHS

3. **If $\tan 2A = \cot (A - 180)$ and 2A is an acute angle find the value of A**

$\tan 2A = \cot (A - 18^0)$

$\Rightarrow \cot(90-2A) = \cot(A-18^0) \Rightarrow 90^0 -2A = A-18^0 \Rightarrow 3A = 108^0 \Rightarrow A = 36^0$

4. **If $\tan A = \cot B$, Prove that $A + B = 90^0$**

LHS $= \tan A = \cot B$

$\Rightarrow \cot(90-A) = \cot B \Rightarrow 90 - A = B \Rightarrow A + B = 90^0$

5. **If $\sec 4A = cosec (A - 20^0)$ and 4A is an acute angle find the value of A**

$\sec 4A = cosec (A - 20^0)$

$\Rightarrow cosec(90 - 4A) = cosec(A - 20^0) \Rightarrow 90 - 4A = A - 20^0 \Rightarrow 5A = 110 \Rightarrow A = 22^0$

6. **If A, B and C are the interior angles of ΔABC then show that $\sin\frac{(B+C)}{2} = \cos\frac{A}{2}$**

Let A , B and C are the interior angles of ΔABC

$\Rightarrow A + B + C = 180^0 \Rightarrow B + C = 180 - A$

$\Rightarrow \frac{B+C}{2} = \frac{180-A}{2} \Rightarrow \frac{B+C}{2} = 90 - \frac{A}{2}$

$\Rightarrow \sin\frac{(B+C)}{2} = \sin\left(90 - \frac{A}{2}\right) \Rightarrow \sin\frac{(B+C)}{2} = \cos\frac{A}{2}$

7. **Express $Sin67^0 + \cos75^0$ in terms of the trigonometric ratios in between 0^0 and 45^0**

$Sin67^0 + \cos75^0$

$= \sin(90-23^0) + \cos(90-15^0) = \cos 23^0 + \sin15^0$

11.5 ತ್ರಿಕೋನಮಿತಿ ನಿತ್ಯಸಮೀಕರಣಗಳು

You may recall that an equation is called an identity w
involved. Similarly, an equation involving trigonor
trigonometric identity, if it is true for all values of th

$Sin^2A + \cos^2A = 1$

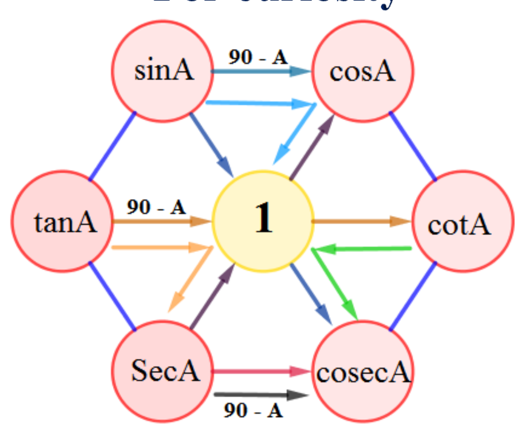
$\tan^2 + 1 = \sec^2A$

$1 + \cot^2A = cosec^2A$

Note: $\frac{\sin A}{\cos A} = \tan A$

$\frac{\cos A}{\sin A} = \cot A$

For curiosity



Example 12 : Express the ratios cos A, tan A and sec A in terms of sin A.

$\cos^2A + \sin^2A = 1 \Rightarrow \cos^2A = 1 - \sin^2A$

$\cos A = \sqrt{1 - \sin2A}$

$\therefore$ tanA $= \frac{\text{sinA}}{\text{cosA}} \Rightarrow \frac{\text{sinA}}{\sqrt{1-\text{sin2A}}}$ $\Rightarrow$secA $= \frac{1}{\text{cosA}} \Rightarrow \frac{1}{\sqrt{1-\text{sin2A}}}$

Example 13 : Prove that sec A (1 – sin A)(sec A + tan A) = 1.

LHS = sec A (1 - sin A)(sec A + tan A)

$= \frac{1}{\text{cosA}}(1 - \sin A)\left(\frac{1}{\text{cosA}} + \frac{\text{sinA}}{\text{cosA}}\right) = \left(\frac{1-\sin A}{\text{cosA}}\right)\left(\frac{1+\text{sinA}}{\text{cosA}}\right) = \frac{1-\sin^2 A}{\cos^2 A} = \frac{\cos^2 A}{\cos^2 A} = 1$

Example 14: Prove that $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\text{cosec } A - 1}{\text{cosec } A + 1}$

$\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\frac{\text{cosA}}{\text{sinA}} - \cos A}{\frac{\text{cosA}}{\text{sinA}} + \cos A} = \frac{\text{cosA}\left(\frac{1}{\text{sinA}} - 1\right)}{\text{cosA}\left(\frac{1}{\text{sinA}} + 1\right)} = = \frac{\frac{1}{\text{sinA}} - 1}{\frac{1}{\text{sinA}} + 1} = \frac{\text{cosec } A - 1}{\text{cosec } A + 1}$

ಉದಾಹರಣೆ 15:Prove that $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$ using the identity $\sec^2\theta = 1 + \tan^2\theta$

$\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta} + \frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} - \frac{1}{\cos\theta}} = \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$

$= \frac{(\tan\theta + \sec\theta) - 1}{(\tan\theta - \sec\theta) + 1} \text{ x } \frac{\tan\theta - \sec\theta}{\tan\theta - \sec\theta} = \frac{(\tan\theta + \sec\theta)(\tan\theta - \sec\theta) - (\tan\theta - \sec\theta)}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$

$= \frac{(\tan^2\theta - \sec^2\theta) - (\tan\theta - \sec\theta)}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)} = \frac{-1 - \tan\theta + \sec\theta)}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$

$= \frac{-1}{(\tan\theta - \sec\theta)} = \frac{1}{\sec\theta - \tan\theta}$

Exercise 11.4

1. **Express the trigonometric ratios sinA, secA ಮತ್ತು tanA in terms of cotA**

$\text{cosec}^2 A - \cot^2 A = 1$

$\Rightarrow \text{cosec}^2 A = 1 + \cot^2 A \Rightarrow \frac{1}{\sin^2 A} = 1 + \cot^2 A \Rightarrow \sin^2 A = \frac{1}{1+\cot^2 A} \Rightarrow \sin A = \frac{\pm 1}{\sqrt{1+\cot^2 A}}$

$\sin^2 A = \frac{1}{1+\cot^2 A} \Rightarrow 1 - \cos^2 A = \frac{1}{1+\cot^2 A} \Rightarrow \cos^2 A = 1 - \frac{1}{1+\cot^2 A} \Rightarrow \cos^2 A = \frac{1+\cot^2 A - 1}{1+\cot^2 A}$

$\Rightarrow \frac{1}{\sec^2 A} = \frac{\cot^2 A}{1+\cot^2 A} \Rightarrow \sec^2 A = \frac{1+\cot^2 A}{\cot^2 A} \Rightarrow \sec A = \frac{\pm\sqrt{1+\cot^2 A}}{\cot A} \Rightarrow \tan A = \frac{1}{\cot A}$

2. Write all the trigonometric ratios ∠A in terms of sec A

$\sec A = \frac{1}{\cos A} \Rightarrow \mathbf{\cos A = \frac{1}{\sec A}}$

$\cos^2 A + \sin^2 A = 1 \Rightarrow \sin^2 A = 1 - \cos^2 A \Rightarrow \sin^2 A = 1 - \frac{1}{\sec^2 A}$

$\Rightarrow \sin^2 A = \frac{\sec^2 A - 1}{\sec^2 A} \Rightarrow \mathbf{\sin A = \frac{\pm\sqrt{\sec^2 A - 1}}{\sec A}}$

$\sin A = \frac{1}{\text{cosec } A} \Rightarrow \text{cosec } A = \frac{1}{\sin A} \Rightarrow \mathbf{\text{cosec } A = \frac{\pm \sec A}{\sqrt{\sec^2 A - 1}}}$

$\sec^2 A - \tan^2 A = 1 \Rightarrow \tan^2 A = \sec^2 A + 1$

$\mathbf{\Rightarrow \tan A = \sqrt{\sec^2 A + 1}}$

$\tan A = \frac{1}{\cot A} \Rightarrow \cot A = \frac{1}{\tan A} \Rightarrow \mathbf{\cot A = \frac{1}{\sqrt{\sec^2 A + 1}}}$

3. **Evaluate:**

i) $\frac{\sin^2 63^0 + \sin^2 27^0}{\cos^2 17^0 + \cos^2 73^0}$ ii) $\sin 25^0 \cos 65^0 + \cos 25^0 \sin 65^0$

i) $\frac{\sin^2 63^0 + \sin^2 27^0}{\cos^2 17^0 + \cos^2 73^0}$

$= \frac{\sin^2(90-27^0) + \sin^2 27^0}{\cos^2(90-73^0) + \cos^2 73^0} = \frac{\cos^2 27^0 + \sin^2 27^0}{\sin^2 73^0 + \cos^2 73^0} = \frac{1}{1} = 1$

ii) $\sin 25^0 \cos 65^0 + \cos 25^0 \sin 65^0$

sin 25° cos 65° + cos 25° sin 65°

= sin(90°-25°) cos 65° + cos(90°-65°) sin 65°

= cos 65° cos 65° + sin 65° sin 65° $= \cos^2 65° + \sin^2 65° = 1$

4. **Choose the correct option and justifyyour choice**

i) $9\sec^2 A - 9\tan^2 A$

A) 1 B) 9 C) 8 D) 0

$9\sec^2 A - 9\tan^2 A$

$= 9(\sec^2 A - \tan^2 A)$

$= 9 \times 1 = 9$ $[\because \sec^2 A - \tan^2 A = 1]$

Ans: B) 9

ii) **$(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \text{cosec}\,\theta) =$**

A) 0 B) 1 C) 2 D) -1

$(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \text{cosec}\,\theta)$

$= \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(1 + \frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}\right)$

$= \frac{\cos\theta + \sin\theta + 1}{\cos\theta} \times \frac{\sin\theta + \cos\theta - 1}{\sin\theta}$

$= \frac{(\cos\theta + \sin\theta)^2 - 1}{\cos\theta . \sin\theta} = \frac{\cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta - 1}{\cos\theta . \sin\theta} = \frac{1 + 2\cos\theta\sin\theta - 1}{\cos\theta . \sin\theta} = \frac{2\cos\theta\sin\theta}{\cos\theta . \sin\theta} = 2$

Ans C) 2

iii) (secA + tanA) (1 - sinA) =

A) secA B) sinA C) cosecA D) cosA

(secA + tanA)(1 - sinA)

(secA + tanA) (1 - sinA)

$= \left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)(1 - \sin A) = \left(\frac{1 + \sin\theta}{\cos\theta}\right)(1 - \sin A) = \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A} = \cos A$

Ans: D) cosA

iv) **$\frac{1 + \tan^2 A}{1 + \cot^2 A} =$**

A) $\sec^2 A$ B) -1 C) $\cot^2 A$ D) $\tan^2 A$

$\frac{1 + \tan^2 A}{1 + \cot^2 A}$

$= \frac{1 + \frac{1}{\cot^2 A}}{1 + \cot^2 A} = \frac{\cot^2 A + 1}{\cot^2 A} \times \frac{1}{1 + \cot^2 A} = \frac{1}{\cot^2 A} = \tan^2 A$

Ans: D) $\tan^2 A$

5. **Prove the following identities, where the angles involved are acute angles for which the expressions are defined.**

i) $(\text{cosec}\,\theta - \cot\theta)^2 = \frac{1 - \cos\theta}{1 + \cos\theta}$

L.H.S. = $(\text{cosec}\,\theta - \cot\theta)^2$

$= (\text{cosec}^2\theta + \cot^2\theta - 2\text{cosec}\,\theta\cot\theta) = \left(\frac{1}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} - \frac{2\cos\theta}{\sin^2\theta}\right)$

$= \left(\frac{1 + \cos^2\theta - 2\cos\theta}{1 - \cos^2\theta}\right) = \frac{(1 - \cos\theta)^2}{(1 + \cos\theta)(1 - \cos\theta)} = \frac{1 - \cos\theta}{1 + \cos\theta}$ RHS

ii) $\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2\sec A$

L.H.S. $= \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A}$

$= \frac{\cos^2 A + (1 + \sin A)^2}{(1 + \sin A)\cos A} = \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{(1 + \sin A)\cos A}$

$= \frac{1 + 1 + 2\sin A}{(1 + \sin A)\cos A} = \frac{2 + 2\sin A}{(1 + \sin A)\cos A} = \frac{2(1 + \sin A)}{(1 + \sin A)\cos A} = \frac{2}{\cos A} = 2\sec A$ = R.H.S.

iii) $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta.\cos\theta$

[Hint : Write the expression in terms of sin and cos]

L.H.S. $=\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}$

$=\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}=\frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$

$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}+\frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$

$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}-\frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$

$=\frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^2\theta}{\cos\theta}-\frac{\cos^2\theta}{\sin\theta}\right]$

$=\frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^3\theta-\cos^3\theta}{\cos\theta.\sin\theta}\right]$

$=\frac{1}{(\sin\theta-\cos\theta)}\left[\frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\cos\theta.\sin\theta}\right]=\left[\frac{(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\cos\theta.\sin\theta}\right]$

$=\left[\frac{1+\sin\theta\cos\theta}{\cos\theta.\sin\theta}\right]=\left[\frac{1}{\cos\theta.\sin\theta}+1\right]$ $=1+\sec\theta\ \text{cosec}\ \theta=$ R.H.S.

iv) $\frac{1+\sec A}{\sec A}=\frac{\sin^2 A}{1-\cos A}=2\sec A$

[Hint: simplify LHS and RHS separately]

L.H.S. $=\frac{1+\sec A}{\sec A}=\frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}=\frac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}}$

$=\frac{\cos A+1}{\cos A}\times\frac{\cos A}{1}=\cos A+1$

R.H.S. $=\frac{\sin^2 A}{1-\cos A}=\frac{(1+\cos A)(1-\cos A)}{1-\cos A}=\cos A+1$

L.H.S. = R.H.S.

v) Prove that $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\text{cosec}\ A+\cot A$ **using the identity** $\text{cosec}^2 A=1+\cot^2 A$

L.H.S. $=\frac{\cos A-\sin A+1}{\cos A+\sin A-1}$

$=\frac{\frac{\cos A-\sin A+1}{\sin A}}{\frac{\cos A+\sin A-1}{\sin A}}=\frac{\cot A-1+\text{cosec}A}{\cot A+1-\text{cosec}A}$ [Divide both denominator and numerator by sin A]

$=\frac{\cot A-\text{cosec}^2A+\cot^2A+\text{cosec}A}{\cot A+1-\text{cosec}A}$ (using $\text{cosec}^2A-\cot^2A=1$)

$=\frac{\cot A+\text{cosec}A-(\text{cosec}^2A-\cot^2A)}{\cot A+1-\text{cosec}A}=\frac{(\cot A+\text{cosec}A)(1-\text{cosec}A-\cot A)}{1-\text{cosec}A+\cot A}$

$=\cot A+\text{cosec}\ A=$ R.H.S.

vi) $\sqrt{\frac{1+\sin A}{1-\sin A}}=\sec A+\tan A$

$=\sqrt{\frac{1+\sin A}{1-\sin A}\times\frac{1+\sin A}{1+\sin A}}=\sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$

$=\sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}=\frac{1+\sin A}{\cos A}=\frac{1}{\cos A}+\frac{\sin A}{\cos A}=\sec A+\tan A=$RHS

vii) $\frac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}=\tan\theta$

L.H.S. $=\frac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}$

$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-1)} = \frac{\sin\theta[1-2(1-\cos^2\theta)]}{\cos\theta(2\cos^2\theta-1)}$

$= \frac{\sin\theta[1-2+2\cos^2\theta]}{\cos\theta(2\cos^2\theta-1)} = \frac{\sin\theta[2\cos^2\theta-1]}{\cos\theta(2\cos^2\theta-1)} = \frac{\sin\theta}{\cos\theta} = \tan\theta$ = R.H.S.

viii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

L.H.S. = $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2\cos A \sec A$

$= (\sin^2 A + \cos^2 A) + 2\sin A\left(\frac{1}{\sin A}\right) + 2\cos A\left(\frac{1}{\cos A}\right) + 1 + \tan^2 A + 1 + \cot^2 A$

$= 1 + 2 + 2 + 2 + \tan^2 A + \cot^2 A = 7 + \tan^2 A + \cot^2 A$ = R.H.S.

ix) $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$

[Hint: simplify LHS and RHS separately]

L.H.S. = $(\operatorname{cosec} A - \sin A)(\sec A - \cos A)$

$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$

$= \left(\frac{\cos^2 A}{\sin A}\right)\left(\frac{\sin^2 A}{\cos A}\right) = \cos A \sin A$

R.H.S. = $\frac{1}{\tan A + \cot A} = \frac{1}{\frac{\sin^2 A + \cos^2 A}{\cos A . \sin A}}$

$= \frac{1}{\frac{1}{\cos A . \sin A}} = \cos A . \sin A$

L.H.S. = R.H.S.

x) $\frac{1+\tan^2 A}{1+\cot^2 A} = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

L.H.S. = $\frac{1+\tan^2 A}{1+\cot^2 A}$

$= \frac{1+\tan^2 A}{1+\frac{1}{\tan^2 A}} = \frac{1+\tan^2 A}{\frac{\tan^2 A + 1}{\tan^2 A}} = \frac{1+\tan^2 A}{\frac{1+\tan^2 A}{\tan^2 A}} = \tan^2 A$

$\left(\frac{1-\tan A}{1-\cot A}\right)^2 = \left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$

$= \left(\frac{1-\tan A}{\frac{\tan A - 1}{\tan A}}\right)^2 = \left(\frac{1-\tan A}{\frac{-(1-\tan A)}{\tan A}}\right)^2 = (-\tan A)^2 = \tan^2 A$

Summery:

1. In right angle triangle ABC, $\angle B = 90^0$

SinA	**$\frac{\text{Opposite side}}{\text{Hypotenuse}}$**
CosA	$\frac{\text{Adjacent side}}{\text{Hypotenuse}}$
Tan A	$\frac{\text{Opposite side}}{\text{Adjacent}}$

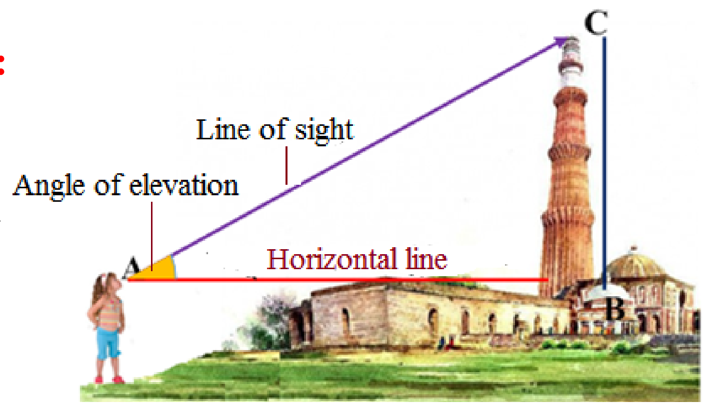
$\frac{1}{\text{SinA}}$	$\frac{\text{Hypotenuse}}{\text{oppsite side}}$	**CosecA**
$\frac{1}{\text{CosA}}$	$\frac{\text{Hypotenuse}}{\text{Adjacent side}}$	**SecA**
$\frac{1}{\text{Tan A}}$	$\frac{\text{Adjacent side}}{\text{Opposite side}}$	**CotA**

3. If one of the trigonometric ratios of an acute angle is known, the remaining trigonometric ratios of the angle can be easily determined.
4. The value of sin A or cos A never exceeds 1, whereas the value of sec A or cosec A is always greater than or equal to 1.
5. sin (90° – A) = cos A, cos (90° – A) = sin A;
   tan (90° – A) = cot A, cot (90° – A) = tan A; s
   ec (90° – A) = cosec A, cosec (90° – A) = sec A
6. $\sec^2 A - \tan^2 A = 1, 0^0 \leq A < 90^0$
   $\text{cosec}^2 A = 1 + \cot^2 A, 0^0 \leq A < 90^0$
   $\sin^2 A + \cos^2 A = 1,$

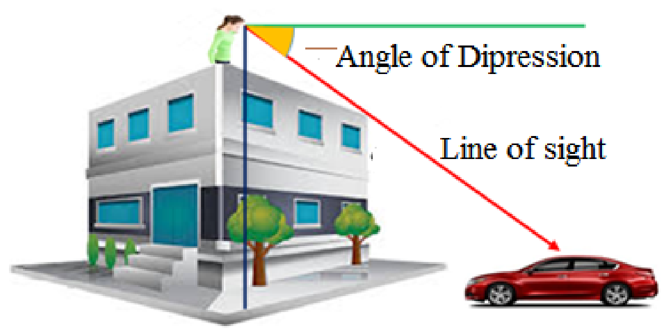
12 Some Applications of Trigonometry

Trigonometry is one of the most ancient subjects studied by scholars all over the world. As we have said in Chapter 11, trigonometry was invented because its need arose in astronomy. Since then the astronomers have used it, for instance, to calculate distances from the Earth to the planets and stars. Trigonometry is also used in geography and in navigation. The knowledge of trigonometry is used to construct maps, determine the position of an island in relation to the longitudes and latitudes. Surveyors have used trigonometry for centuries. One such large surveying project of the nineteenth century was the 'Great Trigonometric Survey' of British India for which the two largest-ever theodolites were built. During the survey in 1852, the highest mountain in the world was discovered. From a distance of over160 km, the peak was observed from six different stations. In 1856, this peak was named after Sir George Everest, who had commissioned and first used the giant theodolites (see the figure alongside). The theodolites are now on display in the Museum of the Survey of India in Dehradun.

12.2 Height and distance:

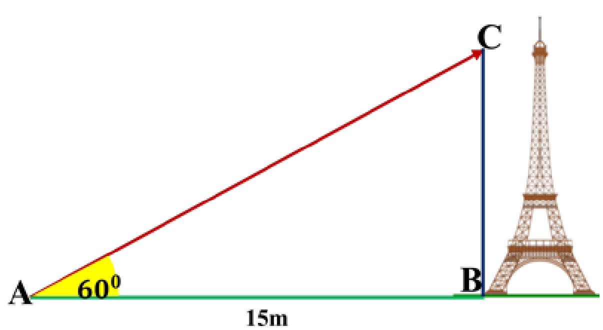


Thus, the line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer. The angle of elevation of the point viewed is the angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level, i.e., the case when we raise our head to look at the object



Thus, the angle of depression of a point on the object being viewed is the angle formed by the line of sight with the horizontal when the point is below the horizontal level, i.e., the case when we lower our head to look at the point being viewed

Example1 : A tower stands vertically on the ground. From a point on the ground, which is 15 m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 60°. Find the height of the tower.



Let Height of the tower = BC; AB = 15m

$\tan 60^0 = \frac{BC}{AB} \Rightarrow \sqrt{3} = \frac{BC}{15} \Rightarrow BC = 15\sqrt{3}$ m

Example 2 : An electrician has to repair an electric fault on a pole of height 5 m. She needs to reach a point 1.3m below the top of the pole to undertake the repair work (see Fig. 12.5). What should be the length of the ladder that she should use which, when inclined at an angle of 60° to the horizontal, would enable her to reach the required position? Also, how far from the foot of the pole should she place the foot of the ladder? (You may take $\sqrt{3}$ = 1.73)

Height of the PoleAD = 5m; The height in which repair work to be done BD = 5 – 1.3 = 3.7m

Height of the Ladder BC = ?.

Distance between the foot of the pole and the foot of the ladder CD=?

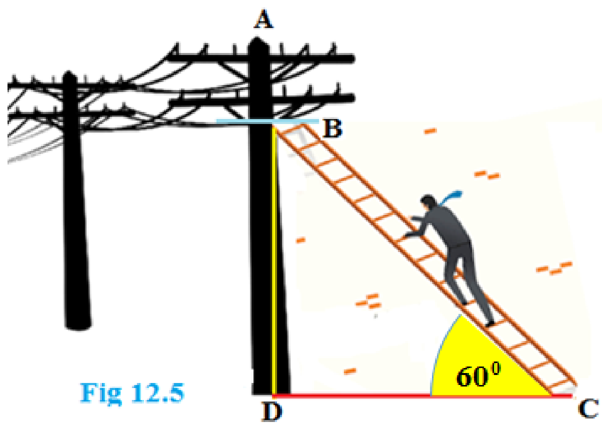


Fig 12.5

$\sin 60^0 = \frac{BD}{BC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{3.7}{BC} \Rightarrow BC = \frac{3.7x2}{\sqrt{3}} = \frac{7.4}{\sqrt{3}}$ m $\approx \frac{740}{173} = 4.28$m

$\text{Tan}60^0 = \frac{BD}{CD} \Rightarrow \sqrt{3} = \frac{3.7}{CD} = \frac{3.7}{\sqrt{3}}$ m ≈ 2.14m

∴ Height of the Ladder BC = 4.28m and Distance between the foot of the pole and the foot of the ladder CD = 2.14m

Example 3 : An observer 1.5 m tall is 28.5 m away from a chimney. The angle of elevation of the top of the chimney from her eyes is 45°. What is the height of the chimney?

Height of the observer CD = BE = 1.5m,
Distance from the chimney to the observer DE = CB = 28.5m;
Height of the chimney AB = ?

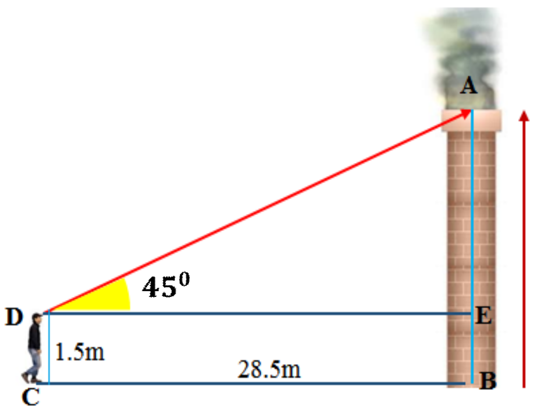
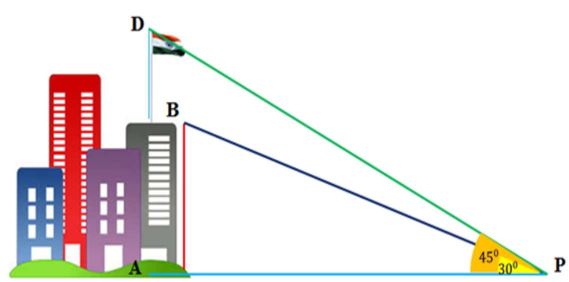


Fig 12.6

$\tan 45^0 = \frac{AE}{DE} \Rightarrow 1 = \frac{AE}{28.5} \Rightarrow AE = 28.5$m

∴ Height of the chimney AB = AE + BE = 28.5 + 1.5 = 30m

Example 4 : From a point P on the ground the angle of elevation of the top of a 10 m tall building is 30°. A flag is hoisted at the top of the building and the angle of elevation of the top of the flagstaff from P is 45°. Find the length of the flagstaff and the distance of the building from the point P. (you take $\sqrt{3}$ = 1.732)



Height of the building AB = 10m

$\text{Tan}30^0 = \frac{AB}{AP} \Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{AP} \Rightarrow AP = 10\sqrt{3} = 10x1.732 = 17.32$m

$\text{Tan}45^0 = \frac{AD}{AP} \Rightarrow 1 = \frac{AD}{17.32} \Rightarrow AD = 17.32$m

∴ Length of the flagstaff = AD – AB = 17.32 – 10 = 7.32m

Example 5 : The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than when it is 60°. Find the height of the tower.

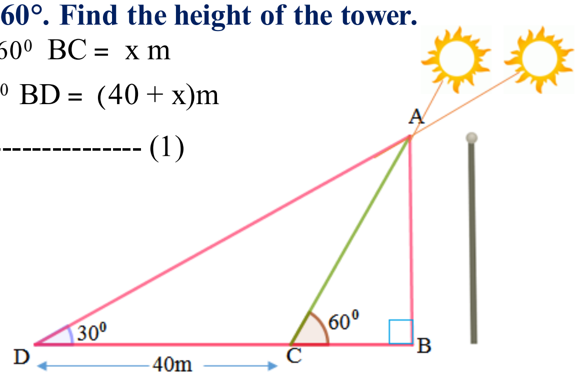


Fig 12.8

Let length of the shadow when Sun's altitude 60^0 BC = x m

∴ length of the shadow when Sun's altitude 30^0 BD = (40 + x)m

$\tan 60^0 = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{AB}{x} \Rightarrow AB = \sqrt{3}x$ -------------------------- (1)

$\tan 30^0 = \frac{AB}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{40+x} \Rightarrow 40+x = \sqrt{3}AB$

$\Rightarrow 40 + x = \sqrt{3} . \sqrt{3}x$

$\Rightarrow 40 + x = 3x$

$\Rightarrow 2x = 40 \Rightarrow x = 20m$

∴ (1) $\Rightarrow AB = \sqrt{3}x \Rightarrow AB = 20\sqrt{3}$ m

Example 6 : The angles of depression of the top and the bottom of an 8 m tall building from the top of a multi-storeyed building are 30° and 45°, respectively. Find the height of the multi-storeyed building and the distance between the two buildings.

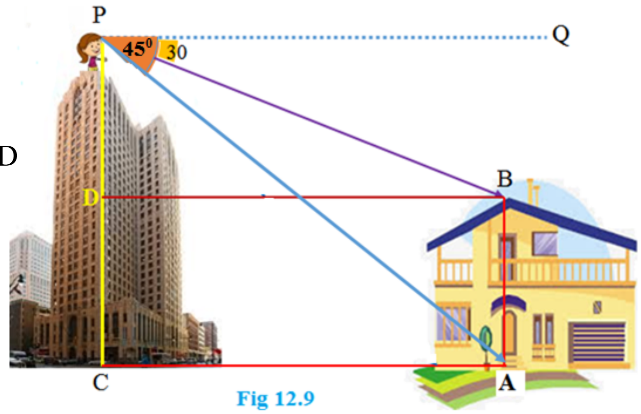


Fig 12.9

Height of the building = AB = 8m

Height of the multi-storeyed building PC = PD + CD = PD + AB = PD + 8m ------------ (1)

Distance between the buildings=AC = BD

PQ∥BD,

∴ ∠BPQ = ∠PBD [Alternate angles]

$\therefore \tan 30^0 = \frac{PD}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{PD}{BD}$

$\Rightarrow BD = \sqrt{3}PD$ ------------------------(2)

PQ∥AC,

∴ ∠APQ = ∠PAC [Alternate angles]

$\therefore \tan 45^0 = \frac{PC}{AC} \Rightarrow 1 = \frac{PD+8}{\sqrt{3}PD}$ [From (1) and (2)]

$\Rightarrow \sqrt{3}PD = PD + 8 \Rightarrow PD(\sqrt{3} - 1) = 8 \Rightarrow PD = \frac{8}{\sqrt{3}-1} = \frac{8(\sqrt{3}+1)}{2} = 4(\sqrt{3}+1)$

∴ Height of the multi-storeyed building PC = PD + 8m $= 4(\sqrt{3}+1) + 8$

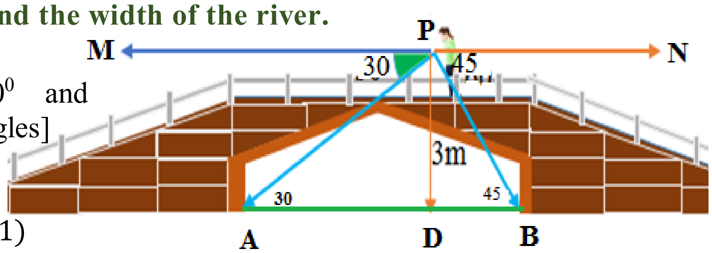
$= 4\sqrt{3} + 12 = 4(3 + \sqrt{3})m$

∴ Distance between the buildings' = Distance between the buildings $= 4(3 + \sqrt{3})m$

[Distance between the buildings = AC = BD ⇒ BD

$= 4\sqrt{3}(\sqrt{3}+1)$ [(2) ರಿಂದ] ⇒ BD $= 4(3 + \sqrt{3})$m]

Example: From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45°, respectively. If the bridge is at a height of 3 m from the banks, find the width of the river.



Width of the river = AD + BD

MN∥AB ⇒ ∠MPA = ∠A = 30^0 and ∠NPD = ∠B = 45^0 [Alternate angles]

$\tan 30^0 = \frac{PD}{AD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{3}{AD}$

$\Rightarrow AD = 3\sqrt{3}$ m ---------------------(1)

$\tan 45^0 = \frac{PD}{BD} \Rightarrow 1 = \frac{3}{BD}$

$\Rightarrow BD = 3m$ --------------------(2)

From (1) and (2)

Width of the river = $AD + BD = 3\sqrt{3} + 3 = 3(\sqrt{3} + 1)m$

Exercise 12.1

A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30° (see Fig. 9.11).

Height of the pole BC

$\text{Sin}30^0 = \frac{BC}{AC} \Rightarrow \frac{1}{2} = \frac{BC}{20} \Rightarrow BC = 10m$

∴ Height BC = 10m

2. **A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.**

Let BC is the broken part of the tree

∴ Total height of the tree = AB+BC

$\cos 30° = \frac{AC}{BC}$

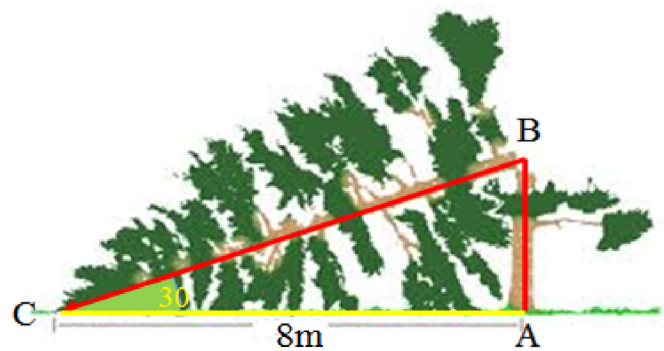
$\Rightarrow \frac{\sqrt{3}}{2} = \frac{8}{BC} \Rightarrow BC = \frac{16}{\sqrt{3}}$

$\tan 30° = \frac{AB}{AC}$

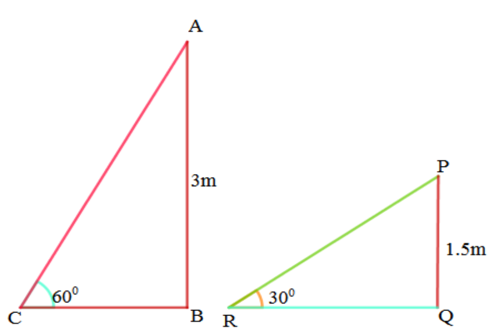
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{8} \Rightarrow AB = \frac{8}{\sqrt{3}}m$

∴ Height of the tree

$= AB + BC = \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} = \frac{24}{\sqrt{3}}m$



3. **A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3 m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?**



Let the length of the side making inclination 60^0 = AC and

Length of the slide making inclination $30°$ = PR

According to question,

In right angle triangle ΔABC,

$\sin 30° = \frac{PQ}{PR} \Rightarrow \frac{1}{2} = \frac{1.5}{PR} \Rightarrow PR = 3m$

In right angle triangle ΔPQR,

$\sin 60° = \frac{AB}{AC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{3}{AC} \Rightarrow AC = \frac{6}{\sqrt{3}} m = 2\sqrt{3}\ m$

$\therefore$ Length of the slides 3m and $2\sqrt{3}$ m .

4. **The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower**

Let height of the tower = AB

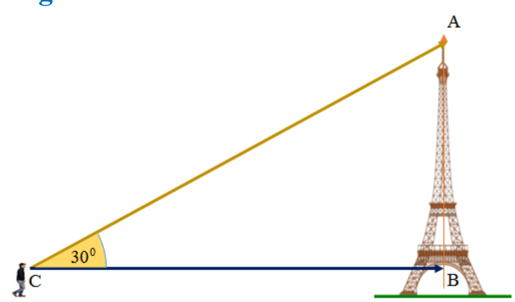
Distance from the foot of the tower to the point BC = 30m

In right angle triangle ΔABC ,

$\tan 30^0 = \frac{AB}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{30}$

$\Rightarrow AB = \frac{30}{\sqrt{3}} = 10\sqrt{3}m$



5. **A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.**

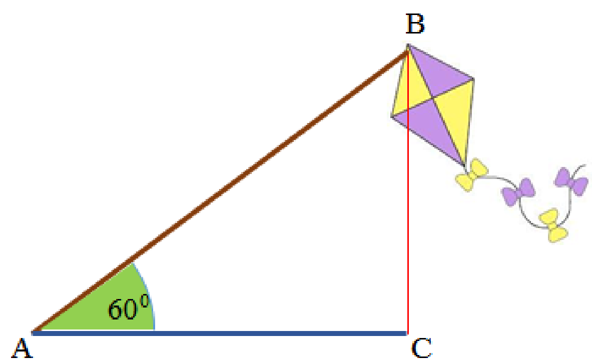
Height of the kite BC = 60m

Length of the tread = AB,

In right angle triangle ΔABC ,

$Sin 60^0 = \frac{BC}{AB} \Rightarrow \frac{\sqrt{3}}{2} = \frac{60}{AB}$

$\Rightarrow AB = \frac{120}{\sqrt{3}} = 40\sqrt{3}m$

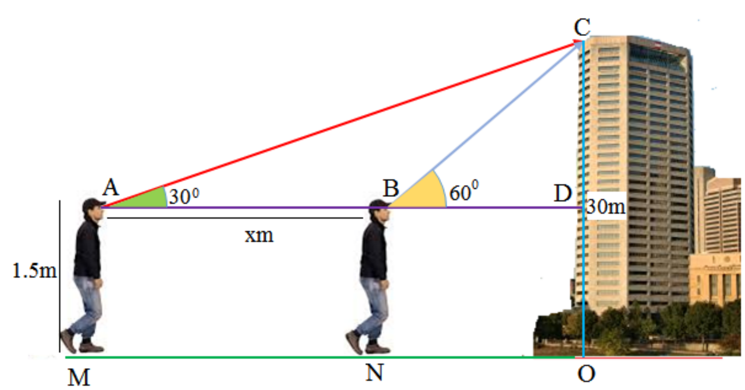


6. **A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building?**

Angle of elevation when the boy is at M = 30^0 After walking x meter, the angle of elevation is 60^0 at N.

$\therefore$ MN = AB = x.

Height of the building = OC = 30 m



CD = OC - OD = (30 - 1.5) = 28.5 m, According to question

In right angle ΔADC, $\tan 30° = \frac{CD}{AD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{28.5}{AD} \Rightarrow AD = 28.5\sqrt{3}$ m

In right angle ΔCBD ಯಲ್ಲಿ, $\tan 60° = \frac{CD}{BD} \Rightarrow \sqrt{3} = \frac{28.5}{BD} \Rightarrow BD = \frac{28.5}{\sqrt{3}} = 9.5\sqrt{3}$ m

∴ MN = AB = x = $(28.5\sqrt{3} - 9.5\sqrt{3}) = 19\sqrt{3}$ m

∴The distance he walked towards the building = $19\sqrt{3}$ m

7. **From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower**

Height of the building = BC= 20 m

A point on the ground where the angle of elevation measured is D

Height of the tansmission tower AB = AC - BC

According to question,

In right angle triangle ΔBCD,

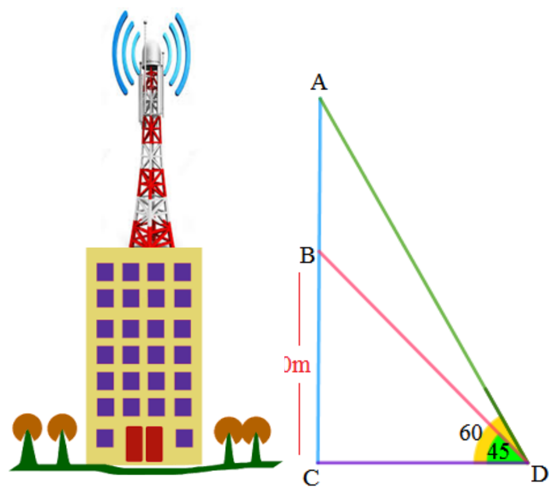
$\tan 45° = \frac{BC}{CD} \Rightarrow 1 = \frac{20}{CD}$

$\Rightarrow$ CD = 20 m

In right angle triangle ΔACD,

$\tan 60° = \frac{AC}{CD} \Rightarrow \sqrt{3} = \frac{AC}{20}$

$\Rightarrow AC = 20\sqrt{3}$ m



Height of the tansmission tower AB = AC - BC = $(20\sqrt{3} - 20)$ m = $20(\sqrt{3} - 1)$ m.

8. **A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.**

Let the height of the statue = AB

The point where the angle of elevation measured is D

Height of the pedestal BC = AC – AB

By question, In right angle ΔBCD ,

$\tan 45° = \frac{BC}{CD} \Rightarrow 1 = \frac{BC}{CD}$

⇒ BC = CD.

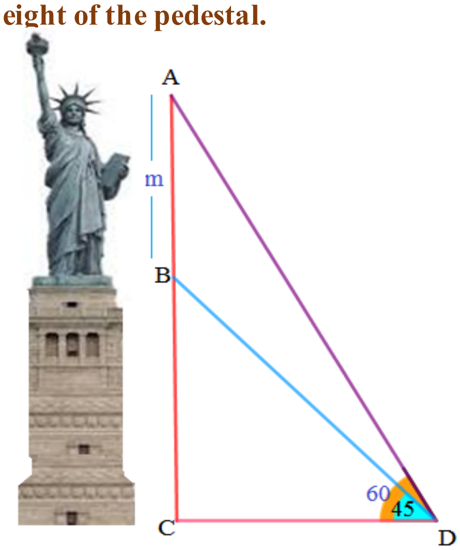
In right angle ΔACD,

$\tan 60° = \frac{AC}{CD}$

$\Rightarrow \sqrt{3}CD = 1.6\text{ m} + BC \Rightarrow \sqrt{3}BC = 1.6\text{ m} + BC$

$\Rightarrow \sqrt{3}BC - BC = 1.6\text{ m} \Rightarrow BC(\sqrt{3} - 1) = 1.6$ m

$\Rightarrow BC(\sqrt{3} - 1) = \frac{1.6}{\sqrt{3} - 1} \Rightarrow BC = 0.8(\sqrt{3} + 1)$ m



∴Height of the pedestal BC = $0.8(\sqrt{3}+1)$ m.

9. **The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.**

Given: the height of the tower CD = 50 m
Let the height of the building = AB
Distance from the foot of the building to the tower = BC
According to question In right angle triangle ΔBCD,

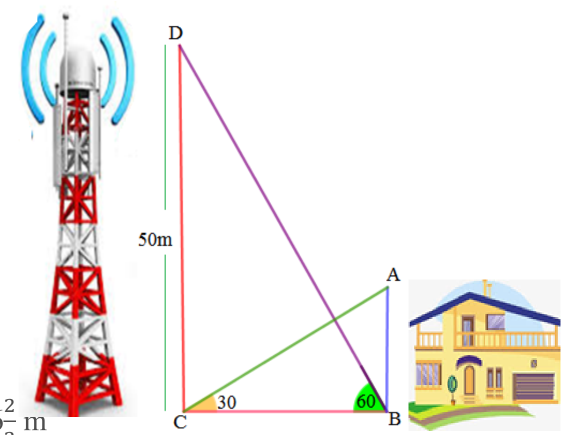
$\tan 60° = \frac{CD}{BC} \Rightarrow \sqrt{3} = \frac{50}{BC} \Rightarrow BC = \frac{50}{\sqrt{3}}$

In right angle triangle ΔABC,

$\tan 30° = \frac{AB}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BC} \Rightarrow AB = \frac{BC}{\sqrt{3}} \Rightarrow AB = \frac{50}{3}m$

Therefore height of the building = $\frac{50}{3}$ m = $16\frac{2}{3}$ m



10. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles.

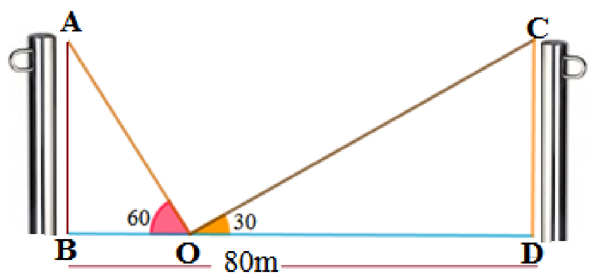
AB and CD are the two poles of equal height
The point on the ground where the angle of elevation is measured is O.

The distance between the poles = BD

According to question,
AB = CD, OB + OD = 80 m
In right angle triangle ΔCDO,

$\tan 30° = \frac{CD}{OD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{CD}{OD}$

$\Rightarrow CD = \frac{OD}{\sqrt{3}}$ ------------------- (1)



In right angle triangle ΔABO,

$\tan 60° = \frac{AB}{OB} \Rightarrow \sqrt{3} = \frac{AB}{80-OD} \Rightarrow AB = \sqrt{3}\,(80\text{-}OD)$

AB = CD [Given]

$\Rightarrow \sqrt{3}\,(80\text{-}OD) = \frac{OD}{\sqrt{3}} \Rightarrow 3(80\text{-}OD) = OD$

$\Rightarrow 240 - 3\,OD = OD \Rightarrow 4\,OD = 240 \Rightarrow OD = 60$

Substitute OD = 60 in (1) we get,

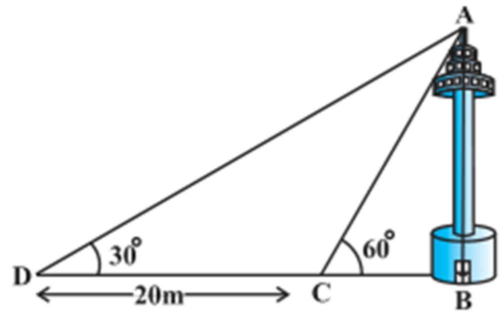
$CD = \frac{60}{\sqrt{3}} \Rightarrow CD = 20\sqrt{3}$ m

$OB + OD = 80 \text{ m} \Rightarrow OB = (80\text{-}60) \text{ m} = 20 \text{ m}$

Therefore the height of the poles = $20\sqrt{3}$ m and the distance from the point of elevation to the poles = 60m and 20m

11. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joing this point to the foot of the tower, the angle of elevation of the top of the tower is 30^0 (see Fig. 12.12). Find the height of the tower and width of the canal.

Let Height of the TV tower = AB; CD = 20 m [Given]



According to question, In triangle ΔABD,

$\tan 30° = \frac{AB}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{CD+BC}$

$\Rightarrow AB = \frac{(20+BC)}{\sqrt{3}}$ -------------------- (1)

In right angle triangle ΔABC,

$\tan 60° = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{AB}{BC}$

$\Rightarrow AB = \sqrt{3}\,BC$ ------------------------- (2)

From equation (1) and (2)

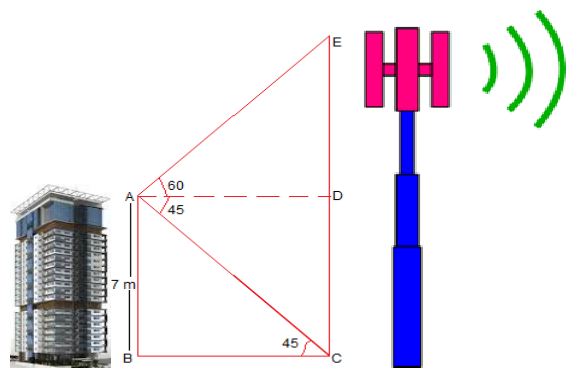
$\frac{(20+BC)}{\sqrt{3}} = \sqrt{3}\,BC \Rightarrow 3\,BC = 20 + BC \Rightarrow 2\,BC = 20 \Rightarrow BC = 10$ m

Substitute BC =10 in equation (2) $AB = 10\sqrt{3}$ m

The height of the tower = $10\sqrt{3}$ m and width of the canal = 10 m.

12. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower

Height of the building AB = 7 m ; Height of the tower = EC



A is the point of elevation

EC = DE + CD

CD = AB = 7 m ಮತ್ತು BC = AD

By question, In right angle triangle ΔABC,

$\tan 45° = \frac{AB}{BC} \Rightarrow 1 = \frac{7}{BC} \Rightarrow BC = 7\text{ m} = AD$

In right angle triangle ΔADE,

$\tan 60° = \frac{DE}{AD} \Rightarrow \sqrt{3} = \frac{DE}{7}$

$\Rightarrow DE = 7\sqrt{3}$ m

Height of the tower = EC = DE + CD

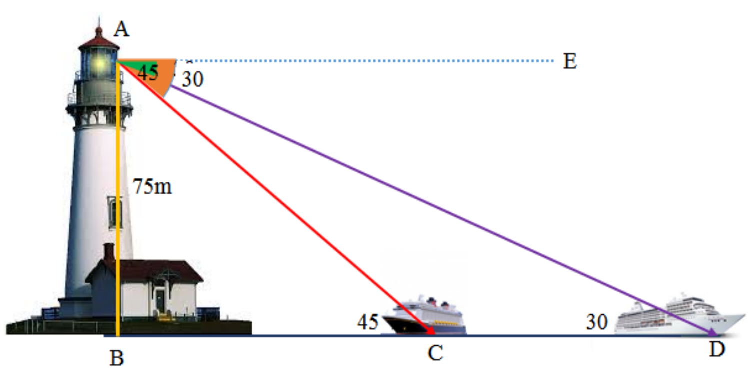
$= (7\sqrt{3} + 7)$ m $= 7\sqrt{3}+1)$ m.

13. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships

Height of the lighthouse AB = 75 m.

Let the positons of the ships C and D

According to question,



In right angle triangle ΔABC,

$\tan 45° = \frac{AB}{BC}$

$\Rightarrow 1 = \frac{75}{BC}$

$\Rightarrow BC = 75$ m

In right angle triangle ΔABD

$\tan 30° = \frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{BD}$

$\Rightarrow BD = 75\sqrt{3}$m

Therefore the distance between the ships = CD = BD - BC

$= (75\sqrt{3} - 75)$m $= 75(\sqrt{3}-1)$m.

14. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30° (see Fig.12.13). Find the distance travelled by the balloon during the interval.

Let the initial position of the balloon and the later position be A and B respectively

The height of the ballon = 88.2 m - 1.2 m = 87 m

The distance travelled by the bolloon

= DE = CE - CD

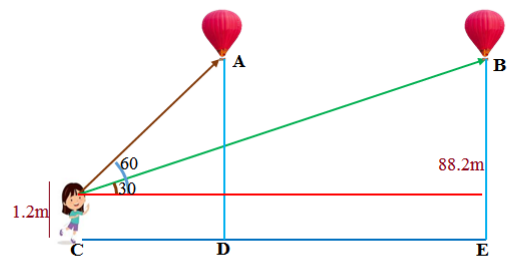
According to question in ΔBEC,

$\tan 30° = \frac{BE}{CE} \Rightarrow \frac{1}{\sqrt{3}} = \frac{87}{CE} \Rightarrow CE = 87\sqrt{3}$ m

In right triangle ΔADC,

$\tan 60° = \frac{AD}{CD} \Rightarrow \sqrt{3} = \frac{87}{CD}$

$\Rightarrow CD = \frac{87}{\sqrt{3}}$ m $= 29\sqrt{3}$ m



∴ The distance travelled by the bolloon DE = CE - CD = $(87\sqrt{3} - 29\sqrt{3})$ m = $58\sqrt{3}$ m.

15. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

Let the height of the tower = AB

D is the initial position of the car and C is the later position

BC is the distance from the car to the tower

According to question, In right triangle ΔABC,

$\tan 60° = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{AB}{BC} \Rightarrow BC = \frac{AB}{\sqrt{3}}$ m

In right triangle ΔADB,

$\tan 30° = \frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BC + CD}$

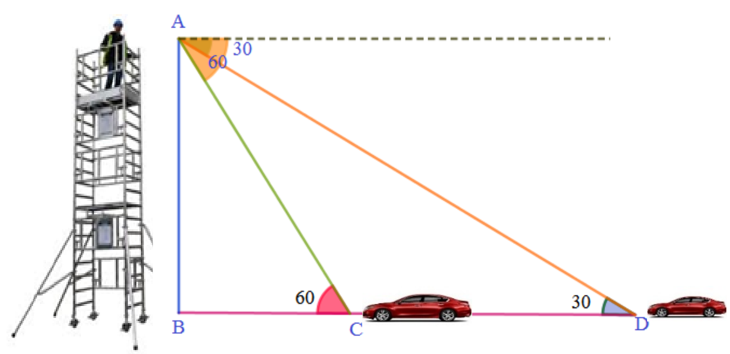
$\Rightarrow AB\sqrt{3} = \frac{AB}{\sqrt{3}} + CD$

$\Rightarrow CD = AB\sqrt{3} - \frac{AB}{\sqrt{3}}$

$\Rightarrow CD = AB\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$

$\Rightarrow CD = AB\left(\frac{3-1}{\sqrt{3}}\right)$

$\Rightarrow CD = \frac{2AB}{\sqrt{3}}$ m



Here, distance BC is half the distance CD

Therefore time taken to move BC is half the time taken to move CD

Given that the time taken by the car to move distance CD = 6 sec.

∴ The time taken to move the distance BC = 6/2 = 3 sec.

16. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m

The height of the tower AB

C and D are the points from the tower at the distance 4 m and 9 m respectively

According to question,

In right triangle ΔADB,

$\tan x = \frac{AB}{BC} \Rightarrow \tan x = \frac{AB}{4}$

$\Rightarrow AB = 4 \tan x$ ----------------- (1)

In right triangle ΔABD,

$\tan (90°-x) = \frac{AB}{BD} \Rightarrow \cot x = \frac{AB}{9}$

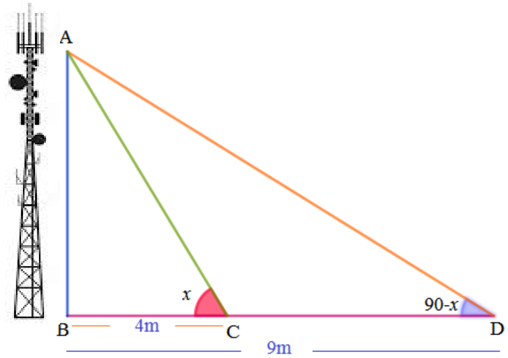
$\Rightarrow AB = 9 \cot x$ ---------------- (2)

Multiplying equation (1) and (2) ,

$AB^2 = 9 \cot x \times 4 \tan x \Rightarrow AB^2 = 36 \Rightarrow AB = \pm 6$

AB = -6 is not possible.

Therefore height of the tower is 6m



Summary:

1. (i) The line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer.
(ii) The angle of elevation of an object viewed, is the angle formed by the line of sight with the horizontal when it is above the horizontal level, i.e., the case when we raise our head to look at the object.
(iii) The angle of depression of an object viewed, is the angle formed by the line of sight with the horizontal when it is below the horizontal level, i.e., the case when we lower our head to look at the object.

The height or length of an object or the distance between two distant objects can be determined with the help of t

13 Statistics

13.2 Mean of Grouped data

Average: $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$ **[i = 1 to n]**

Example 1 : The marks obtained by 30 students of Class X of a certain school in a Mathematics paper consisting of 100 marks are presented in table below. Find the mean of the marks obtained by the students

x	y
10	1
20	1
36	3
40	4
50	3
56	2
60	4
70	4
72	1
80	1
88	2
92	3
95	1

=

x_i	f_i	$x_i f_i$
10	1	10
20	1	20
36	3	108
40	4	160
50	3	150
56	2	112
60	4	240
70	4	280
72	1	72
80	1	80
88	2	176
92	3	276
95	1	96
	$\sum f_i = 30$	$\sum x_i f_i = 1779$

Average $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

$\frac{1779}{30} = 59.53$

Direct Method to find average:

C.I.	No.of students
10–25	2
25–40	3
40–55	7
55–70	6
70–85	6
85–100	6

Class Interval	(f_i)	Mid-point (x_i)	$f_i x_i$
10–25	2	17.5	35.0
25–40	3	32.5	97.5
40–55	7	47.5	332.5
55–70	6	62.5	375.0
70–85	6	77.5	465.0
85–100	6	92.5	555.0
	$\sum f_i = 30$		$\sum f_i x_i = 1860$

Average $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1860}{30} = 62$

Assumed Mean Method:

$d_i = x_i - a$ [Here, a = 47.5]

Class Interval	(f_i)	Mid-point (x_i)	$d_i = x_i - 47.5$	$f_i d_i$
10–25	2	17.5	–30	–60
25–40	3	32.5	–15	–45
40–55	7	47.5	0	0
55–70	6	62.5	15	90
70–85	6	77.5	30	182
85–100	6	92.5	45	270
	$\sum f_i = 30$			$\sum f_i d_i = 435$

Average $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i} = 47.5 + \frac{435}{30} = 47.5 + 14.5 = 62$

Step Deviation Method:

$d_i = x_i - a$ [Here, a = 47.5] and h = 15

Class Interval	(f_i)	Mid-point (x_i)	$u_i = \frac{x_i - 47.5}{15}$	$f_i u_i$
10–25	2	17.5	–2	–4
25–40	3	32.5	–1	–3
40–55	7	47.5	0	0
55–70	6	62.5	1	6
70–85	6	77.5	2	12
85–100	6	92.5	3	18
	$\sum f_i = 30$			$\sum f_i u_i = 29$

Average $\bar{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h = 47.5 + \frac{29}{30} \times 15 = 47.5 + \frac{29}{2} = 47.5 + 14.5 = 62$

Note: If all *di* have common multiple then step deviation method is the best method

We get the same average in all three methods.

Assumed Mean and step deviation methods are the simplified form of Direct Method.

Example 2 : The table below gives the percentage distribution of female teachers in the primary schools of rural areas of various states and union territories (U.T.) of India. Find the mean percentage of female teachers by all the three methods discussed

a = 50, h = 10

Percentage of Female teachers	f_i
15–25	6
25–35	11
35–45	7
45–55	4
55–65	4
65–75	2
75–85	1

C.I.	f_i	x_i	$d_i = x_i - 50$	$u_i = \frac{x_i - 20}{10}$	$f_i x_i$	$f_i d_i$	$f_i u_i$
15–25	6	20	–30	–3	120	–180	–18
25–35	11	30	–20	–2	330	–220	–22
35–45	7	40	–10	–1	280	–70	–7
45–55	4	50	0	0	200	0	0
55–65	4	60	10	1	240	40	4
65–75	2	70	20	2	140	40	4
75–85	1	80	30	3	80	30	3
	$\sum f_i = 35$				1390	–360	–36

From the above table $\sum f_i = 35$, $\sum f_i x_i = 1390$, $\sum f_i d_i = -360$, $\sum f_i u_i = -36$

Average in direct Method $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1390}{35} = 39.71$

Average in Assumed Mean Method $\bar{x}$ =a + $\frac{\sum f_i d_i}{\sum f_i} = 50 - \frac{360}{35} = 50 - 10.29 = 39.71$

Average in step deviation Method $\bar{x}$ = a + $\frac{\sum f_i u_i}{\sum f_i}$ x h=50 $- \frac{36}{35}$ x 10 = 50 – 10.29= 39.71

Example 3 : The distribution below shows the number of wickets taken by bowlers in one-day cricket matches. Find the mean number of wickets by choosing a suitable method. What does the mean signify?

No.of Wickets	No.of Bowlers
20 –60	7
60 –100	5
100 –150	16
150 –250	12
250 –350	2
350 –450	3

C.I.	f_i	x_i	$d_i = x_i - 200$	$u_i = \frac{x_i - 200}{10}$	$f_i u_i$
20 –60	7	40	–160	–8	–56
60 –100	5	80	–120	–6	–30
100 –150	16	125	–75	–3.75	–60
150 –250	12	200	0	0	0
250 –350	2	300	100	5	10
350 –450	3	400	200	10	30
	45				–106

$\bar{x}$ = a + $\frac{\sum f_i u_i}{\sum f_i}$ x h = 200 – $\frac{106}{45}$ x 20 = 200 – 47.11 = 152.89

Exercise – 13.1

1. A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house
Which method did you use for finding the mean, and why?

a = 7 , h = 2

No.of Plants	No.of Houses
0–2	1
2–4	2
4–6	1
6–8	5
8–10	6
10–12	2
12–14	3

C.I.	f_i	x_i	$d_i = x_i - 7$	$u_i = \frac{x_i - 20}{2}$	$f_i x_i$	$f_i d_i$	$f_i u_i$
0–2	1	1	–6	–3	1	–6	–3
2–4	2	3	–4	–2	6	–8	–4
4–6	1	5	–2	–1	5	–2	–1
6–8	5	7	0	0	35	0	0
8–10	6	9	2	1	54	12	6
10–12	2	11	4	2	22	8	4
12–14	3	13	6	3	39	18	9
	$\sum f_i$ =20			0	162	22	11

From the above table $\sum f_i = 35$, $\sum f_i x_i = 162$, $\sum f_i d_i = 20$, $\sum f_i u_i = 11$

Average from Direct Method $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1620}{20} = 8.1$

Average from assumed Mean Method $\bar{x}$ =a + $\frac{\sum f_i d_i}{\sum f_i} = 7 + \frac{22}{20} = 7 + 1.1 = 8.1$

Average from step deviation method $\bar{x}$ = a + $\frac{\sum f_i u_i}{\sum f_i}$ x h = 7 + $\frac{11}{20}$ x 2 = 7 + 1.1 = 8.1

[**You can use any method. Because of simple tabulation we can use direct method here**]

2. Consider the following distribution of daily wages of 50 workers of a factory. Find the mean daily wages of the workers of the factory by using an appropriate method

a = 75.5, h = 3

Daily wages (Rs)	No.of workers
100–120	12
120–140	14
140–160	8
160–180	6
180–200	10

C.I.	f_i	x_i	$d_i = x_i - 150$	$u_i = \frac{x_i - 150}{20}$	$f_i x_i$	$f_i d_i$	$f_i u_i$
100–120	12	110	−40	−2	1320	−480	−24
120–140	14	130	−20	−1	1820	−280	−14
140–160	8	150	0	0	1200	0	0
160–180	6	170	20	1	1020	120	6
180–200	10	190	40	2	1900	400	20
	50				7260	−240	−12

From the above table $\sum f_i = 50$, $\sum f_i x_i = 7260$, $\sum f_i d_i = -240$, $\sum f_i u_i = -12$

Average from Direct Method $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{7260}{50} = 145.2$

Average from assumed Mean Method $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i} = 150 + \frac{-240}{50} = 150 - 4.8 = 145.2$

Average from step deviation method $\bar{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h = 150 + \frac{-12}{50} \times 20 = 150 - 4.8 = 145.2$

[**Can use any method. But Assumed mean method is more suitable here**]

3. The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is ` 18. Find the missing frequency f

Daily Pocket allowences(Rs)	11–13	13–15	15–17	17–19	19–21	21–23	23–25
No.of Children	7	6	9	13	f	5	4

a = 18 , h = 2

C.I.	f_i	x_i	$d_i = x_i - 18$	$u_i = \frac{x_i - 18}{2}$	$f_i x_i$	$f_i d_i$	$f_i u_i$
11–13	7	12	−6	−3	84	−42	−21
13–15	6	14	−4	−2	84	−24	−12
15–17	9	16	−2	−1	144	−18	−9
17–19	13	18	0	0	234	0	0
19–21	f	20	2	1	20f	2f	1f
21–23	5	22	4	2	110	20	10
23–25	4	24	6	3	96	24	12
	$\sum f_i = 44 + f$				752+20f	−40+2f	−20+ f

From the above table $\sum f_i = 44 + f$, $\sum f_i x_i = 752 + 20f$, $\sum f_i d_i = -40 + 2f$, $\sum f_i u_i = -20 + f$

Average from Direct Method $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

$18 = \frac{752 + 20f}{44 + f} \Rightarrow 18(44 + f) = 752 + 20f$

$\Rightarrow 792 + 18f = 752 + 20f \Rightarrow 40 = 2f \Rightarrow f = 20$

Average from assumed Mean Method $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$

$18 = 18 + \frac{-40 + 2f}{44 + f} \Rightarrow 0 = (-40 + 2f) \Rightarrow 2f = 40 \Rightarrow f = 20$

Average from step deviation method $\bar{x}$ = a + $\frac{\sum f_i u_i}{\sum f_i}$ x h

$\Rightarrow 18 = 18 + \frac{-20+f}{44+f} \times 20 \Rightarrow -20 + f = 0 \Rightarrow f = 20$

[Wecan use any method here]

4. Thirty women were examined in a hospital by a doctor and the number of heartbeats per minute were recorded and summarised as follows. Find the mean heartbeats per minute for these women, choosing a suitable method

No.of Heart beats/Minute	65−68	68−71	71−74	74−77	77−80	80−83	83−86
No.of women	2	4	3	8	7	4	2

a = 75.5 , h = 3

C.I.	f_i	x_i	$d_i = x_i - 75.5$	$u_i = \frac{x_i - 75.5}{3}$	$f_i d_i$	$f_i u_i$
65−68	2	66.5	−9	−3	−18	−6
68−71	4	69.5	−6	−2	−24	−8
71−74	3	72.5	−3	−1	−9	−3
74−77	8	75.5	0	0	0	0
77−80	7	78.5	3	1	21	7
80−83	4	81.5	6	2	24	8
83−86	2	84.5	9	3	18	6
	$\sum f_i$ =30				12	4

From the above table $\sum f_i = 30, \sum f_i d_i = 12, \sum f_i u_i = 4$

Average from assumed Mean Method $\bar{x}$ = a + $\frac{\sum f_i u_i}{\sum f_i}$ = 75.5 + $\frac{12}{30}$ = 75.5 + 0.4 = 75.9

Average from step deviation method $\bar{x}$ = a + $\frac{\sum f_i u_i}{\sum f_i}$ x h = 75.5 + $\frac{4}{30}$ x 3 = 75.5 + 0.4 = 75.9

[Direct method is not suitable here]

5. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes.

No.of Mangoes	50−52	53−55	56−58	59−61	62−64
No.of boxes	15	110	135	115	25

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

a = 57 , h = 3

C.I.	f_i	x_i	$d_i = x_i - 150$	$u_i = \frac{x_i - 75.5}{3}$	$f_i d_i$	$f_i u_i$
50−52	15	51	−6	−2	−90	−30
53−55	110	54	−3	−1	−330	−110
56−58	135	57	0	0	0	0
59−61	115	60	3	1	345	115
62−64	25	63	6	2	150	50
	$\sum f_i$ =400				75	25

Average from assumed Mean Method $\bar{x}$ =a + $\frac{\sum d_i x_i}{\sum f_i}$

$= 57 + \frac{75}{400} = 57 + 0.1875 = 57.1875 \approx 57.19$

Average from step deviation method $\bar{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$

$= 57 + \frac{25}{400} \times 3 = 57 + 0.1875 = 57.1875 \approx 57.19$

Here, Assumed mean method is more suitable

6. The table below shows the daily expenditure on food of 25 households in a locality

Daily expenditure(Rs)	100–150	150–200	200–250	250–300	300–350
No.of households	4	5	12	2	2

Find the mean daily expenditure on food by a suitable method.

a = 225 , h = 50

C.I.	f_i	x_i	$d_i = x_i - 150$	$u_i = \frac{x_i - 75.5}{3}$	$f_i d_i$	$f_i u_i$
100–150	4	125	–100	–2	–400	–8
150–200	5	175	–50	–1	–250	–5
200–250	12	225	0	0	0	0
250–300	2	275	50	1	100	2
300–350	2	325	100	2	200	4
	$\sum f_i = 25$				–350	–7

Average from assumed Mean Method $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$

$= 225 + \frac{-350}{25} = 225 - 14 = 211$

Average from step deviation method $\bar{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$

$= 225 + \frac{-7}{25} \times 50 = 225 - 14 = 211$

For this problem step deviation method is more suitable

7. To find out the concentration of SO_2 in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below
Find the mean concentration of SO_2 in the air

Concentration of SO_2	Freequency
0.00 – 0.04	4
0.04 – 0.08	9
0.08 – 0.12	9
0.12 – 0.16	2
0.16 – 0.20	4
0.20 – 0.24	2

C.I.	f_i	x_i	$f_i x_i$
0.00 – 0.04	4	0.02	0.08
0.04 – 0.08	9	0.06	0.54
0.08 – 0.12	9	0.10	0.90
0.12 – 0.16	2	0.14	0.28
0.16 – 0.20	4	0.18	0.72
0.20 – 0.24	2	0.22	0.44
	$\sum f_i = 30$		2.96

Average from Direct Method $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{2.96}{30} = 0.099$ppm

The mean concentration of SO_2 in the air = 0.099ppm

8. A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent

No.of days	No.of students
0–6	11
6–10	10
10–14	7
14–20	4
20–28	4
28–38	3
38–40	1

C.I.	f_i	x_i	$f_i x_i$
0–6	11	3	33
6–10	10	8	80
10–14	7	12	84
14–20	4	17	68
20–28	4	24	96
28–38	3	33	99
38–40	1	39	39
	$\sum f_i$ =40		499

From the above table $\sum f_i = 40$, $\sum f_i x_i = 499$,

Average from Direct Method $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{499}{40} = 12.475$

9. The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

Literacy rate(%)	No.of cities
45–55	3
55–65	10
65–75	11
75–85	8
85–95	3

C.I.	f_i	x_i	$f_i x_i$	$d_i = x_i - 70$	$f_i d_i$
45–55	3	50	150	–20	–60
55–65	10	60	600	–10	–100
65–75	11	70	770	0	0
75–85	8	80	640	10	80
85–95	3	90	270	20	60
	$\sum f_i$ =35		2430	0	–20

From the above table $\sum f_i = 35$, $\sum f_i x_i = 2430$,

Average from Direct Method $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{2430}{35} = 69.43$

Average from Assumed mean method $\bar{x} = a + \frac{\sum d_i x_i}{\sum f_i} = 70 + \frac{-20}{35} = 60.43$

13.3 Mode of Grouped Data

A mode is that value among the observations which occurs most often, that is, the value of the observation having the maximum frequency

Example: 4 The wickets taken by a bowler in 10 cricket matches are as follows:

2 6 4 5 0 2 1 3 2 3

Find the mode of the data

No.of wickets	0	1	2	3	4	5	6
No.of matches	1	1	3	2	1	1	1

Clearly, 2 is the number of wickets taken by the bowler in the maximum number (i.e., 3) of matches. So, the mode of this data is 2

Mode $= l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

L = **lower limit of the modal class**
h = **size of the class interval (assuming all class sizes to be equal),**
f_1 = **frequency of the modal class,**
f_0 = **frequency of the class preceding the modal class,**
f_2 = **frequency of the class succeeding the modal class**

Example 5 : A survey conducted on 20 households in a locality by a group of students resulted in the following frequency table for the number of family members in a household

Family size	1 - 3	3 - 5	5 - 7	7 - 9	9 - 11
No.of families	7	8	2	2	1

Find the mode of this data

Here the maximum class frequency is 8, and the class corresponding to this frequency is 3 – 5. So, the modal class is 3 – 5

modal class = 3 – 5, lower limit (l) of modal class = 3, class size (h) = 2

frequency (f_1) of the modal class = 8,

frequency (f_0) of class preceding the modal class $f_0 = 7$

frequency (f_2) of class succeeding the modal class $f_2 = 2$

Now substitute the values in the formula:

Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \text{x } h = 3 + \left[\frac{8-7}{2(8)-7-2}\right] \text{x } 2$

$= 3 + \left[\frac{1}{16-9}\right] \text{x } 2 = 3 + \frac{2}{7} = 3.286$

∴ Therefore, the mode of the data above is 3.286.

Example 6 : The marks distribution of 30 students in a mathematics examination are given in Table 13.3 of Example 1. Find the mode of this data. Also compare and interpret the mode and the mean.

Class Intervals	10–25	25–40	40–55	55–70	70–85	85–100
No.of students	2	3	7	6	6	6

Refer the table 13.3 of example . Maximum students are in the class interval 40-45, it is the modal class,

∴ $l = 40$, h = 15, $f_1 = 7$, $f_0 = 3$, $f_2 = 6$

Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \text{x } h$

Mode = $40 + \left[\frac{7-3}{2(7)-3-6}\right] \text{x } 15 = 40 + \left[\frac{4}{14-9}\right] \text{x } 15$

$= 40 + \frac{4}{5} \text{ x } 15 = 40 + 12$

∴ The mode of the given data is 52

Exercise 13.2

1. **The following table shows the ages of the patients admitted in a hospital during a year:**

Age(in years)	5 – 15	15–25	25–35	35–45	45–55	55–65
No.of patients	6	11	21	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

Maximum number of patients =23

Therefore 35-45 is the modal class interval

∴ $l = 35$, h = 10, $f_1 = 23$, $f_0 = 21$, $f_2 = 14$

Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

Mode = $35 + \left[\frac{23 - 21}{2(23) - 21 - 14}\right] \times 10 = 35 + \left[\frac{2}{46 - 35}\right] \times 10 = 35 + \frac{2}{11} \times 10 = 35 + 1.81$

∴ The mode of the above data is 36.81

(a = 30 , h = 10)

C.I.	f_i	x_i	$d_i = x_i - 70$	$u_i = \frac{x_i - 30}{10}$	$f_i u_i$
5–15	6	10	–20	–2	–12
15–25	11	20	–10	–1	–11
25–35	21	30	0	0	0
35–45	23	40	10	1	23
45–55	14	50	20	2	28
55–65	5	60	30	3	15
	$\sum f_i = 80$				43

By step deviation method $\bar{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h = 30 + \frac{43}{80} \times 10 = 30 + 5.375 = 35.375$

So, we conclude that maximum number of patients admitted in the hospital are of the age 36.81 years(Approx) whereas the average age of the patient admitted in the hospital is 35.375years

2. The following data gives the information on the observed lifetimes (in hours) of 225 electrical components

Life time(in hours)	0 – 20	20–40	40–60	60–80	80–100	100–120
Freequency	10	35	52	61	38	29

Determine the modal lifetimes of the components

Maximum frequency =61

It is in the class interval 60 – 80. So, 60 - 80 is the modal class interval.

∴ $l = 60$, h = 20, $f_1 = 61$, $f_0 = 52$, $f_2 = 38$

Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

Mode = $60 + \left[\frac{61 - 52}{2(61) - 52 - 38}\right] \times 20 = 60 + \left[\frac{9}{122 - 90}\right] \times 20$

$= 60 + \frac{9}{32} \times 20 = 60 + 5.625 = 65.625$

∴ The mode of the above given data = 65.625

3. The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure.

Expenditure (in Rs)	No.of families
1000 – 1500	24
1500 – 2000	40
2000 – 2500	33
2500 – 3000	28
3000 – 3500	30
3500 – 4000	22
4000 – 4500	16
4500 – 5000	7

Maximum frequency = 40

Therefore the modal class interval is (1500 – 2000)

∴ $l = 1500$, h = 500, $f_1 = 40$, $f_0 = 24$, $f_2 = 33$

Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

Mode = $1500 + \left[\frac{40 - 24}{2(40) - 24 - 33}\right] \times 500 = 1500 + \left[\frac{16}{80 - 57}\right] \times 500$

$= 1500 + \frac{16}{23} \times 500 = 1500 + 347.83 = 1847.83$

∴ The mode of the given data = 1847.83

C.I.	f_i	x_i	$d_i = x_i - 2750$	$u_i = \frac{x_i - 2750}{500}$	$f_i u_i$
1000 – 1500	24	1250	–1500	–3	–72
1500 – 2000	40	1750	–1000	–2	–80
2000 – 2500	33	2250	–500	–1	–33
2500 – 3000	28	2750	0	0	0
3000 – 3500	30	3250	500	1	30
3500 – 4000	22	3750	1000	2	44
4000 – 4500	16	4250	1500	3	48
4500 – 5000	7	4750	2000	4	28
	$\sum f_i$ =200				-35

By step deviation method $\bar{x}$ = a + $\frac{\sum f_i u_i}{\sum f_i}$ x h

= 2750 + $\frac{-35}{200}$ x 500 = 2750 – 87.5 = 2662.5

4. **The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret the two measures**

No.of students per teacher	No.of state/U.Ts
15 – 20	3
20 – 25	8
25 – 30	9
30 – 35	10
35 – 40	3
40 – 45	0
45 – 50	0
50 – 55	2

Maximum freeqency = 10, of the class interval 30 - 35

Therefore 30 – 35 is the modal class interval

∴ l = 30, h = 5, f_1 = 10, f_0 = 9, f_2 = 3

Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right]$x h

Mode = 30 + $\left[\frac{10-9}{2(10)-9-3}\right]$x 5

= 30 + $\left[\frac{1}{20-12}\right]$x 5

= 30 + $\frac{1}{8}$ x 5 = 30 + 0.625 = 30.625

∴ The mode of the above data is 30.625

C.I.	f_i	x_i	$d_i = x_i - 32.5$	$u_i = \frac{x_i - 32.5}{5}$	$f_i u_i$
15 – 20	3	17.5	–15	–3	–9
20 – 25	8	22.5	–10	–2	–16
25 – 30	9	27.5	–5	–1	–9
30 – 35	10	32.5	0	0	0
35 – 40	3	37.5	5	1	3
40 – 45	0	42.5	10	2	0
45 – 50	0	47.5	15	3	0
50 – 55	2	52.5	20	4	8
	$\sum f_i$ =35				–23

By step deviation Method $\bar{x}$ = a + $\frac{\sum f_i u_i}{\sum f_i}$ x h = 32.5 + $\frac{-23}{35}$ x 5 = 32.5 – 3.29 = 29.21

The students – teacher ratio is 30.625 and average ratio is 29.21

5. **The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches**
Find the mode of the data.

Maximum freequency = 18. It is in the class interval 4000 - 5000

Runs scored	No.of Batsman
3000 - 4000	4
4000 - 5000	18
5000 - 6000	9
6000 - 7000	7
7000 - 8000	6
8000 - 9000	3
9000 - 10000	1
10000 - 11000	1

Therefore 4000 -5000 is the modal class interval

∴ $l = 4000$, h = 1000, $f_1 = 18, f_0 = 4, f_2 = 9$

Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

Mode = $4000 + \left[\frac{18-4}{2(18)-4-9}\right] \times 1000 =$

$4000 + \left[\frac{14}{36-13}\right] \times 1000$

$= 4000 + \frac{14}{23} \times 1000 = 4000 + 608.7 = 4608.7$

∴ The mode of the above data is 4608.7

6. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data

No.of cars	0 – 10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
Freequency	7	14	13	12	20	11	15	8

Maximum freequency = 20. It is in the class interval 40 - 50

Therefore 40 – 50 is the modal class interval

∴ $l = 40$, h = 10, $f_1 = 20$, $f_0 = 12$, $f_2 = 11$

Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

Mode = $40 + \left[\frac{20-12}{2(20)-12-11}\right] \times 10 = 40 + \left[\frac{8}{40-23}\right] \times 10$

$= 40 + \frac{8}{17} \times 10 = 40 + 4.71 = 44.71$

∴ Mode of the given data 44.71

13.4 Median of Grouped Data

the median is a measure of central tendency which gives the value of the middle-most observation in the data. Recall that for finding the median of ungrouped data, we first arrange the data values of the observations in ascending order, then, if n is odd, then the meadian is $\left(\frac{n+1}{2}\right)$th observation and if n is an even, then the dedian is the average of $\left(\frac{n}{2}\right)$ and $\left(\frac{n}{2}+1\right)$ th observation.

After finding the median class, we use the following formula for calculating the median.

Median of Grouped Data

Median = $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

l = lower limit of median class,
n = number of observations
cf = cumulative frequency of class preceding the median class,.
f = frequency of median class
h = class size (assuming class size to be equal).

Example 7 : A survey regarding the heights (in cm) of 51 girls of Class X of a school was conducted and the following data was obtained
Find the median height.

Heights (in cm)	No.of Girls
Less than 140	4
Less than 145	11
Less than 150	29
Less than 155	40
Less than 160	46
Less than 165	51

C.I.	f	cf
Less than 140	4	4
140 – 145	7	11
145 – 150	18	29
150 – 155	11	40
155 – 160	6	46
160 – 165	5	51

Now, n = 51, ∴ $\frac{n}{2}$ = 25.5 It is in the class interval 145 – 150

∴ l (lower limit) = 145, cf = 11. f = 18, h = 5

Median = $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

Median = $145 + \left[\frac{25.5 - 11}{18}\right] \times 5 = 145 + \left[\frac{72.5}{18}\right] = 149.03$

Therefore median of the given data is 149.03

Example 8 : The median of the following data is 525. Find the values of x and y, if the total frequency is 100.

Class interval	Freequency
0 –100	2
100–200	5
200–300	x
300–400	12
400–500	17
500–600	20
600–700	y
700–800	9
800–900	7
900–1000	4

C.I.	f	cf
0 –100	2	2
100–200	5	7
200–300	x	7+x
300–400	12	19+x
400–500	17	36+x
500–600	20	56+x
600–700	y	56+x+y
700–800	9	65+x+y
800–900	7	72+x+y
900–1000	4	76+x+y

Here, n = 100 ∴ 76 + x + y = 100

So, x + y = 24 .... (1)

Median is 525, which is lies in the class interval 500 – 600

∴ l = 500, f = 20, cf = 36 + x, h = 100

Median = $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

$525 = 500 + \left[\frac{50 - 36 - x}{20}\right] \times 100$

525 = 500 + [14 – x] x 5

25 = 70 – 5x ⇒ 5x = 70 – 25 ⇒ 5x = 45 ∴ x = 9

From equation (1) 9 + y = 24 ⇒ y = 15

Remarks: There is a empirical relationship between the three measures of central tendency

3 Median = Mode + 2 average

Exercise 13.3

1. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.

Now, n = 68, $\therefore \frac{n}{2} = 34$ It is in the class interval 125 - 145.

$\therefore$ l = 125, cf = 22, f = 20, h = 20

Median = $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

median = $125 + \left[\frac{34 - 22}{20}\right] \times 20$

= $125 + \left[\frac{12}{20}\right] \times 20 = 125 + 12 = 137$units

Therefore median is 137units

ಮಾಸಿಕ ಬಳಕೆ (ಯೂನಿಟ್‌ಗಳಲ್ಲಿ)	ಗ್ರಾಹಕರ ಸಂಖ್ಯೆ	ಸಂಚಿತ ಆವೃತ್ತಿ
65 – 85	4	4
85 – 105	5	9
105 – 125	13	22
125 – 145	20	42
145 – 165	14	56
165 – 185	8	64
185 – 205	4	68

Average:

C.I.	f_i	x_i	$d_i = x_i - 135$	$u_i = \frac{x_i - 135}{20}$	$f_i d_i$
65 – 85	4	75	–60	–3	–12
85 – 105	5	95	–40	–2	–10
105 – 125	13	115	-20	-1	-13
125 – 145	20	135	0	0	0
145 – 165	14	155	20	1	14
165 – 185	8	175	40	2	16
185 – 205	4	195	60	3	12
	$\sum f_i = 68$				7

By step deviation method $\bar{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h = 135 + \frac{7}{68} \times 20 = 135 + 2.1 = 137.05$

Mode: maximum freequency = 20, which lies in the class interval 125 – 145.

Therefore 125-145 is the modal class interval

$\therefore$ l = 125, h = 20, $f_1 = 20$, $f_0 = 13$, $f_2 = 14$

Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

= $125 + \left[\frac{20 - 13}{2(20) - 13 - 14}\right] \times 20 = 125 + \left[\frac{7}{40 - 27}\right] \times 20 = 125 + \frac{7}{13} \times 20 = 125 + 10.77 = 135.77$

$\therefore$ Therefore mode of the given data is 135.77

So, we conclude that three measures are approximately same.

2. If the median of the distribution given below is 28.5, find the values of x and y

Total freequency = 45 + x + y ⇒ 60 = 45 + x + y

⇒x + y = 15 ---------------------------(1)

Now, n = 60,

$\therefore \frac{n}{2} = 30$ this is in the class interval 20 – 30

$\therefore$ l = 20 .cf = 5 + x, f = 20, h = 10

Median = $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

$28.5 = 20 + \left[\frac{30 - (5 + x)}{20}\right] \times 10$

Class interval	Freequency	cf
0 - 10	5	5
10 - 20	x	5+x
20 - 30	20	25+x
30 - 40	15	40+x
40 - 50	y	40+x+y
50 - 60	5	45+x+y
Total	60	

$8.5 \times 20 = (30 - 5 - x)10 \Rightarrow 170 = 250 - 10x \Rightarrow 10x = 80 \Rightarrow x = 8$

Substitute x = 8 in equation (1),

$\Rightarrow 8 + y = 15 \Rightarrow y = 7$

Therefore x = 8 and y = 7

3. **A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 year.**

Age(in years)	Cumulative freequency
Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

C.I.	f	cf
15–20	2	2
20–25	4	6
25–30	18	24
30–35	21	45
35–40	33	78
40–45	11	89
45–50	3	92
50–55	6	98
55–60	2	100

Totoa frequency = 100

Now, n = 100, $\therefore \frac{n}{2} = 50$ This is in the class interval 35 - 40

So, $l = 35$, cf = 45, f = 33, h = 5

$$\text{Median} = l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$$

$$= 35 + \left[\frac{50 - 45}{33}\right] \times 5 = 35 + \left[\frac{5}{33}\right] \times 5 = 35 + \frac{25}{33} = 35 + 0.76$$

Median = 35.76

4. **The lengths of 40 leaves of a plant are measured correct to the nearest millimetre, and the data obtained is represented in the following table . Find the median length of the leaves.**

(Hint : The data needs to be converted to continuous classes for finding the median, since th e formula assumes continuous classes. The classes then change to 117.5 - 126.5, 126.5 - 135.5, ..., 171.5 - 180.5.]

Length(in mm)	No.of Leaves
118 - 126	3
127 - 135	5
136 - 144	9
145 - 153	12
154 - 162	5
163 - 171	4
172 - 180	2

C.I.	*f*	*cf*
117.5 – 126.5	3	3
126.5 – 135.5	5	8
135.5 – 144.5	9	17
144.5 – 153.5	12	29
153.5 – 162.5	5	34
162.5 – 171.5	4	38
171.5 – 180.5	2	40

Now, n = 40, $\therefore \frac{n}{2} = 20$ This is in the class interval 144.5 – 153.5

So, l = 144.5, cf = 17, f = 12, h = 9

$$\text{Median} = l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$$

$= 144.5 + \left[\frac{20-17}{12}\right] \times 9 = 144.5 + \left[\frac{3}{12}\right] \times 9 = 144.5 + \frac{27}{12} = 144.5 + 2.25 = 146.75$mm

5. The following table gives the distribution of the life time of 400 neon lamps . Find the median life time of a lamp.

Life time in hours	No.of Lamps
1500-2000	14
2000-2500	56
2500-3000	60
3000-3500	86
3500-4000	74
4000-4500	62
4500-5000	48

C.I.	f	cf
1500-2000	14	14
2000-2500	56	70
2500-3000	60	130
3000-3500	86	216
3500-4000	74	290
4000-4500	62	352
4500-5000	48	400

Total freequencies = 400

Now, n = 400, $\therefore \frac{n}{2} = 200$ this is in the class interval 3000 – 3500

Now, l = 3000, cf = 130, f = 86, h = 500

Median= $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h = 3000 + \left[\frac{200-130}{86}\right] \times 500 = 3000 + \left[\frac{70}{86}\right] \times 500$

$= 3000 + 406.98 = 3406.98$

6. 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows

No.of Letters	1 - 4	4 - 7	7 - 10	10 - 13	13 - 16	16 - 19
No.of surnames	6	30	40	16	4	4

Determine the median number of letters in the surnames. Find the mean number of letters in the surnames? Also, find the modal size of the surnames

C.I.	f	cf
1-4	6	6
4-7	30	36
7-10	40	76
10-13	16	92
13-16	4	96
16-19	4	100

Total freequencies = 100

Now, n = 100, $\therefore \frac{n}{2} = 50$ this is in the class interval 7 – 10

So, l = 7, cf = 36, f = 40, h = 3

Median = $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

$7 + \left[\frac{50-36}{40}\right] \times 3 = 7 + \left[\frac{14}{40}\right] \times 3 = 7 + 1.05 = 8.05$

To find the average:

[a = 8.5, h = 3]

C.I.	f_i	x_i	$d_i = x_i - 135$	$u_i = \frac{x_i - 135}{20}$	$f_i d_i$
1-4	6	2.5	-6	-2	-12
4-7	30	5.5	-3	-1	-30
7-10	40	8.5	0	0	0
10-13	16	11.5	3	1	16
13-16	4	14.5	6	2	8
16-19	4	17.5	9	3	12
	$\sum f_i = 100$				-6

By step deviation method $\bar{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$

$= 8.5 + \frac{-6}{100} \times 3 = 8.5 - 0.18 = 8.32$

To find the mode:

Maximum freequency = 40, Which is in the class interval 7 - 10

Therefore the modal class interval is 7 - 10

$\therefore l = 7$, $h = 3$, $f_1 = 30, f_0 = 30$, $f_2 = 16$

Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

$= 7 + \left[\frac{40 - 30}{2(40) - 30 - 16}\right] \times 3 = 7 + \left[\frac{10}{80 - 46}\right] \times 3 = 7 + \frac{10}{34} \times 3 = 7 + 0.88 = 7.88$

$\therefore$ The mode of the given data is 7.88

7. The distribution below gives the weights of 30 students of a class. Find the median weight of the students

Weight in Kgs	40-45	45-50	50-55	55-60	60-65	65-70	70-75
No.of students	2	3	8	6	6	3	2

C.I.	f	cf
40-45	2	2
45-50	3	5
50-55	8	13
55-60	6	19
60-65	6	25
65-70	3	28
70-75	2	30

Total freequencies = 30

Now, n = 30, $\therefore \frac{n}{2} = 15$

which is in the class interval 55 – 60

So, $l = 55$, cf = 13, $f = 6$, h = 5

Meadian = $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

$= 55 + \left[\frac{15 - 13}{6}\right] \times 5 = 7 + \left[\frac{2}{6}\right] \times 5$

Median= 55 + 1.67 = 56.67kg

13.5 Graphical Representation of Cumulative Frequency Distribution

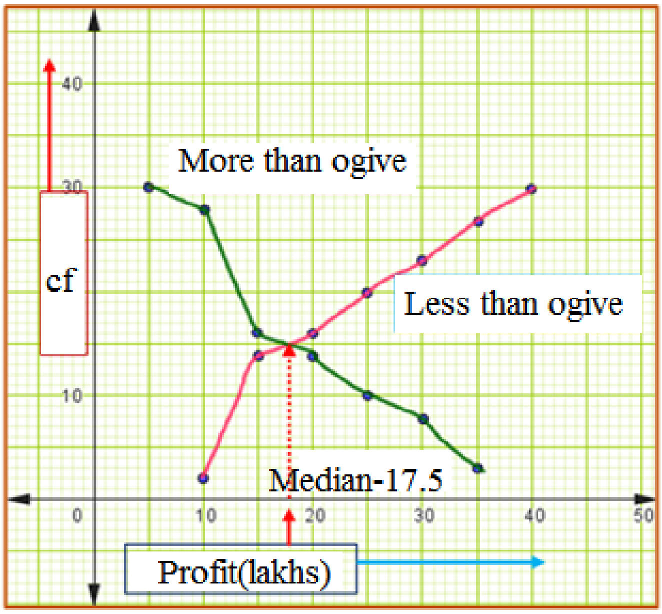
Example 9 : The annual profits earned by 30 shops of a shopping complex in a locality give rise to the following distribution. draw its ogive. Hence obtain the median profit.

Profit (in lakhs)	No.of shopes(f)
More than or equal to 5	30
More than or equal to 10	28
More than or equal to 15	16
More than or equal to 20	14
More than or equal to 25	10
More than or equal to 30	7
More than or equal to 35	3

We first draw the coordinate axes, with lower limits of the profit along the horizontal axis, and the cumulative frequency along the vertical axes. Then, we plot the points (5, 30), (10, 28), (15, 16), (20, 14), (25, 10), (30, 7) and (35, 3). We join these points with a smooth curve to get the 'more than' ogive, as shown in Fig.

Now, let us obtain the classes, their frequencies and the cumulative frequency from the table above

C.I.	5-10	10-15	15-20	20-25	25-30	30-35	35-40
f	2	12	2	4	3	4	3
cf	2	14	16	20	23	27	30



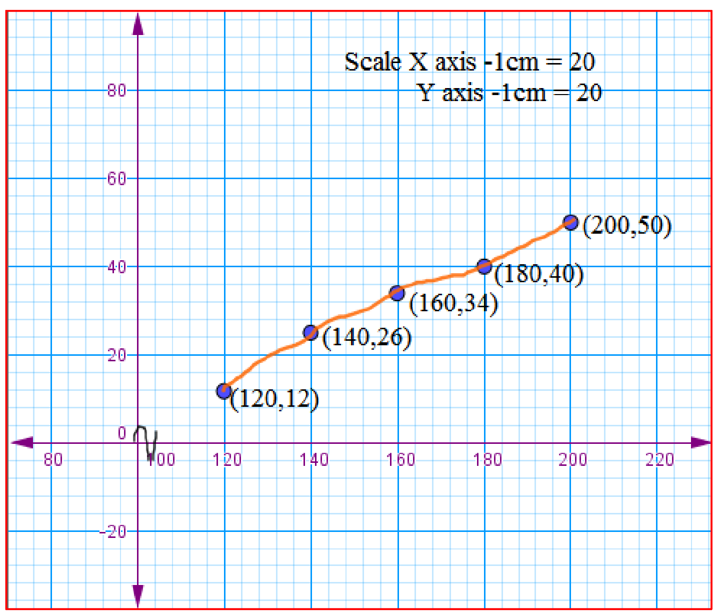
Exercise 13.4

1. **The following table gives the distribution of the life time of 400 neon lamps :**

Daily income(Rs)	100-120	120-140	140-160	160-180	180-200
No.of workers	12	14	8	6	10

Convert the distribution above to a less than type cumulative frequency distribution, and draw its ogive.

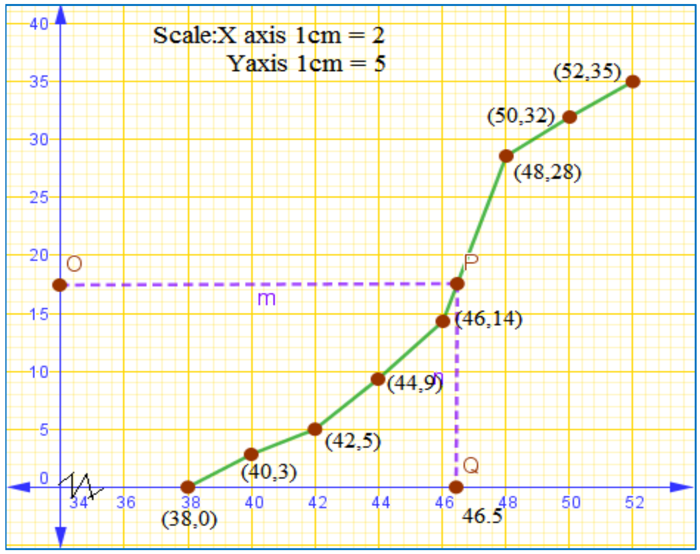
Daily income (Rs)	No.of workers	*cf*
100–120	12	12
120–140	14	26
140–160	8	34
160–180	6	40
180–200	10	50



2. **During the medical check-up of 35 students of a class, their weights were recorded as follows:**
Draw a less than type ogive for the given data. Hence obtain the median weight from the graph and verify the result by using the formula

Weight (in kgs)	38	40	42	44	46	48	50	52
No of students	0	3	5	9	14	28	32	35

Weight (in kgs)	No of students
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

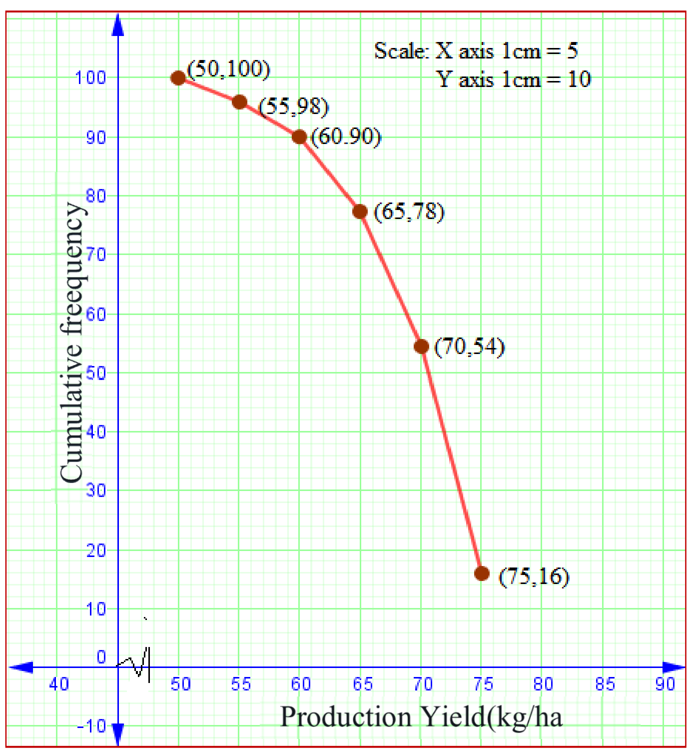


3. The following table gives production yield per hectare of wheat of 100 farms of a village.

Production Yield(kg/ha)	50-55	55-60	60-65	65-70	70-75	75-80
No of farms	2	8	12	24	38	16

Change the distribution to a more than type distribution, and draw its ogive.

Production Yield(kg/ha)	*f*	*cf*
50	2	100
55	8	98
60	12	90
65	24	78
70	38	54
75	16	16



Summary:

1. The mean for grouped data can be found by :

   Direct Method $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

   Assumed mean method: $\bar{x}$ =a + $\frac{\Sigma f_i d_i}{\Sigma f_i}$

   Step deviation method: $\bar{x}$ = a + $\frac{\Sigma f_i u_i}{\Sigma f_i}$ x h

   with the assumption that the frequency of a class is centred at its mid-point, called its class mark

2. The mode for grouped data can be found by using the formula:

   Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

   Where symbols have the meanings

3. The cumulative frequency of a class is the frequency obtained by adding the frequencies of all the classes preceding the given class

4. The median for grouped data is formed by using the formula:.

   meadian = $l + \left[\frac{\frac{n}{2} - cf}{f}\right]$ x h

   Where symbols have the meanings

5. Representing a cumulative frequency distribution graphically as a cumulative frequency curve, or an ogive of the less than type and of the more than type.

6. The median of grouped data can be obtained graphically as the x-coordinate of the point of intersection of the two ogives for this data.

14 Probabilty

14.2 Probability — A Theoretical Approach

Suppose a coin is tossed at random the coin can only land in one of two possible ways — either head up or tail up. suppose we throw a die once. For us, a die will always mean a fair die. They are 1, 2, 3, 4, 5, 6. Each number has the same possibility of showing up.

When we speak of a coin, we assume it to be 'fair', that is, it is symmetrical so that there is no reason for it to come down more often on one side than the other. We call this property of the coin as being 'unbiased'. By the phrase 'random toss', we mean that the coin is allowed to fall freely without any bias or interference

The experimental or empirical probability P(E) of an event E as

$$P(E) = \frac{\text{Number of trials in which the event happened}}{\text{Total number of trails}}$$

The theoretical probability (also called classical probability) of an event E, written as P(E), is defined as

$$P(E) = \frac{\text{No of outcomes favarable to E}}{\text{No.of all possible outcomes of the experiment}}$$

Example 1 : Find the probability of getting a head when a coin is tossed once. Also find the probability of getting a tail.

Random experiment: Tossing a coin once

S - { Tossing a coin once};

S - {H, T} [Here, H – Head T – Tail] – n(S) = 2

A - { Getting Head }- n(A) = 1

$P(A) = \frac{n(A)}{n(S)} = \frac{1}{2}$

B - { Getting Tail }- n(B) = 1

$P(B) = \frac{n(B)}{n(S)} = \frac{1}{2}$

Example 2 : A bag contains a red ball, a blue ball and a yellow ball, all the balls being of the same size. Kritika takes out a ball from the bag without looking into it. What is the probability that she takes out the

(i) Yellow ball (ii) Red ball (iii) Blue ball

S – {Total balls in a bag } ⇒ n(S) = 3

A – { Krthika picking up yellow ball} – n(A) = 1

$P(A) = \frac{n(A)}{n(S)} = \frac{1}{3}$

B – { Krthika picking up red ball} – n(B) = 1

$P(B) = \frac{n(B)}{n(S)} = \frac{1}{3}$

C – { Krthika picking up blue ball} – n(C) = 1

$P(C) = \frac{n(C)}{n(S)} = \frac{1}{3}$

Observe that the sum of the probabilities of all the elementary events of an experiment is 1

Example 3 : Suppose we throw a die once. (i) What is the probability of getting a number greater than 4 ? (ii) What is the probability of getting a number less than or equal to 4 ?

S – {Throwing a dice once } –{ 1,2,3,4,5,6}⇒ n(S) = 6

A – { Getting number more than 4 } – {5,6} - n(A) = 2

$P(A) = \frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

B – { Getting a number equal or less than 4} – {1,2,3,4} - n(B) = 4

$P(B) = \frac{n(B)}{n(S)} = \frac{4}{6} = \frac{2}{3}$

$P(A) = 1 - P(\bar{A})$: where A is an event and $\bar{A}$ is complement of an event A

That is, the probability of an event which is impossible to occur is 0. Such an event is called an **impossible** event

Example: We know that there are only six possible outcomes in a single throw of a die. These outcomes are 1, 2, 3, 4, 5 and 6. Since no face of the die is marked 8, so there is no outcome favourable to 8, i.e., the number of such outcomes is zero. In other words,

getting 8 in a single throw of a die, is impossible

So, the probability of an event which is sure (or certain) to occur is 1. Such an event is called a **sure event** or a certain event.

Example: Since every face of a die is marked with a number less than 7, it is sure that we will always get a number less than 7 when it is thrown once. So, the number of favourable outcomes is the same as the number of all possible outcomes, which is 6.

$0 \le P(E) \le 1$

Example 4 : One card is drawn from a well-shuffled deck of 52 cards. Calculate the probability that the card will
(i) Be an aace (ii) Not be an ace

(i) S – {Picking a card from a deck of 52}

n(S) = 52

E – { The picked card is an ace}

P(E) = 4 [There are 4 aces in a deck of 52]

$P(A) = \frac{n(E)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

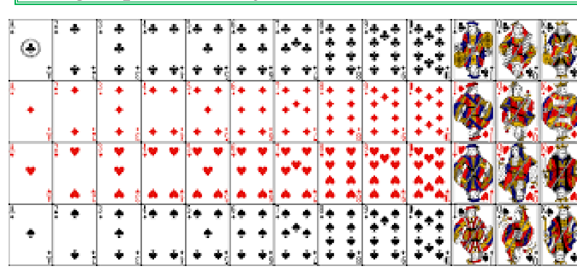
(ii) F – {The card picked is not an ace}

n(F) = 48

$P(F) = \frac{n(F)}{n(S)} = \frac{48}{52} = \frac{11}{13}$

or $P(F) = P(\bar{E}) = 1 - p(E) = 1 - \frac{1}{13} = \frac{11}{13}$

Now, let us take an example related to playing cards. Have you seen a deck of playing cards? It consists of 52 cards which are divided into 4 suits of 13 cards each— spades, hearts, diamonds and clubs. Clubs and spades are of black colour, while hearts and diamonds are of red colour. The cards in each suit are ace, king, queen, jack, 10, 9, 8, 7, 6, 5, 4, 3 and 2. Kings, queens and jacks are called face cards



Example 5 : Two players, Sangeeta and Reshma, play a tennis match. It is known that the probability of Sangeeta winning the match is 0.62. What is the probability of Reshma winning the match?

The probability that Savith wins the match = P(A) = 0.62

The probability that Reshma wins the match $P(\bar{A}) = 1 - P(A) = 1 - 0.62 = 0.38$

Example 6 : Savita and Hamida are friends. What is the probability that both will have (i) different birthdays? (ii) the same birthday? (ignoring a leap year)

(i) Favarable days that Savitha and Hamida have different birthdays $365 - 1 = 364$

Probabilty of having different birthdays P(A) $= \frac{364}{365}$

Probabilty of having same birthday$P(\bar{A}) = \frac{1}{365}$ $[P(\bar{A}) = 1 - P(A)]$

Example 7 : There are 40 students in Class X of a school of whom 25 are girls and 15 are boys. The class teacher has to select one student as a class representative. She writes the name of each student on a separate card, the cards being identical. Then she puts cards in a bag and stirs them thoroughly. She then draws one card from the bag. What is the probability that the name written on the card is the name of (i) a girl? (ii) a boy?

Total number of students: n(S) = 40

Number of Girls – n(A) = 25
Number of boys – n(B) = 15

The probability of drawn card with the name of a Girl $P(A) = \frac{n(A)}{n(S)} = \frac{25}{40} = \frac{5}{8}$

The probability of drawn card with the name of a Boy$P(B) = \frac{n(B)}{n(S)} = \frac{15}{40} = \frac{3}{8}$

OR $P(B) = 1 - P(A) = 1 - \frac{5}{8} = \frac{3}{8}$

Example 8 : A box contains 3 blue, 2 white, and 4 red marbles. If a marble is drawn at random from the box, what is the probability that it will be (i) white (ii) blue (iii) red

The number of marbles in a box = n(S) = 9

The probability of getting white marble $P(W) = \frac{2}{9}$

The probability of getting white blue $P(B) = \frac{3}{9}$

The probability of getting white red $P(B) = \frac{4}{9}$

Example 9 : Harpreet tosses two different coins simultaneously (say, one is of ` 1 and other of ` 2). What is the probability that she gets at least one head?

The two different coins are tossed, the outcomes are S = { HH, HT, TH, TT }⇒ n(S) = 4
The favorable outcomes to get atleast one head – {HT, TH, TT}

Therefore the probability of getting atleast one head – $\frac{3}{4}$

[Example 10 and 11 are not solved because they are optional]

Example 12 : A carton consists of 100 shirts of which 88 are good, 8 have minor defects and 4 have major defects. Jimmy, a trader, will only accept the shirts which are good, but Sujatha, another trader, will only reject the shirts which have major defects. One shirt is drawn at random from the carton. What is the probability that

(i) it is acceptable to Jimmy? (ii) it is acceptable to Sujatha?

Total number of shirts = n(S) = 100
The number of good shirts = 88
(i) The number of outcomes favourable (i.e., acceptable) to Jimmy = 88
Therefore, P (shirt is acceptable to Jimmy) $= \frac{88}{100} = 0.88$
(ii) The number of outcomes favourable to Sujatha = 88 + 8 = 96
So, P (shirt is acceptable to Sujatha) = $\frac{96}{100} = 0.96$

Example 13 : Two dice, one blue and one grey, are thrown at the same time. Write down all the possible outcomes. What is the probability that the sum of the two numbers appearing on the top of the dice is (i) 8 (ii) 13 (iii) less than or equal to 12

The total number of outcomes when two dice are thrown at the same time
(1,1), (1,2), (1,3), (1,4),(1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)

n(S) = 6x6 = 36

(i) A – The sum of two numbers be 8

A – { (2,6), (3,5), (4,4), (5,3), (6,2)} - n(A) = 5

∴ The probability of getting the sum of two numbers be 8 = $\frac{5}{36}$

(ii) B - The sum of two numbers be 13 - n(B) = 0

∴ The probability of getting the sum of two numbers be 13 = $\frac{0}{36} = 0$

(iii) C - B - The sum of two numbers be equal or less than 12

∴ The probability of getting the sum of two numbers be equal or less than = $\frac{36}{36} = 1$

Exercise 14.1

1. **Complete the following statements**

(i) Probability of an event E + Probability of the event 'not E' = ______________

(ii) The probability of an event that cannot happen is _______Such an event is called _______ .

(iii)The probability of an event that is certain to happen is _______ Such an event is called _______.

(iv)The sum of the probabilities of all the elementary events of an experiment is _______.

(v) The probability of an event is greater than or equal to _______ and less than or equal to _______

Answers:

(i) 1 (ii) 0 , impossible event (iii) 1, Sure (iv) 1 (v) 0 , 1

2. **Which of the following experiments have equally likely outcomes? Explain.**

(i) A driver attempts to start a car. The car starts or does not start

(ii) A player attempts to shoot a basketball. She/he shoots or misses the shot.

(iii) A trial is made to answer a true-false question. The answer is right or wrong

(iv) A baby is born. It is a boy or a girl.

Answer

(i) It does not have equally likely outcomes as it depends on various reasons like mechanical problems, fuels etc.

(ii) It does not have equally likely outcomes as it depends on the player how he/she shoots.

(iii) It has equally likely outcomes.

(iv)It has equally likely outcomes.

3. **Why is tossing a coin considered to be a fair way of deciding which team should get the ball at the beginning of a football game?**

Yes, tossing of a coin is a fair way of deciding which team should get the ball at the beginning of a football game because it has only two outcomes either head or tail. A coin is always unbiased

4. **Which of the following cannot be the probability of an event?**
A) $\frac{2}{3}$ B) -1.5 C) 15% D) 0.7 3
The probability of an event is always greater than or equal to 0 and less than or equal to 1.
Thus, (B) -1.5 cannot be the probability of an event.

5. **If P(E) = 0.05, what is the probability of 'not E'?**
The probability of 'not E'=1 - P(E) = 1 – 0.05 = 0.95

6. **A bag contains lemon flavoured candies only. Malini takes out one candy without looking into the bag. What is the probability that she takes out**
(i) an orange flavoured candy? (ii) a lemon flavoured candy?
Answer
(i) Since the bag contains only lemon flavoured.
Therefore, No. of orange flavoured candies = 0
Probability of taking out orange flavoured candies = $\frac{0}{1}$ = 0
(ii) The bag only have lemon flavoured candies.
Probability of taking out lemon flavoured candies = $\frac{1}{1}$ = 1

7. **It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?**
Answer
Let E be the event of having the same birthday. P(E) = 0.992
⇒ P(E) + P(not E) = 1 ⇒ P(not E) = 1 – P(E) ⇒ 1 - 0.992 = 0.008
The probability that the 2 students have the same birthday is 0.008

8. **A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is (i) red? (ii) not red?**
Total number of balls in a bag = n(S) = 3 + 5 = 8
(i) Number of red balls = n(A) = 3
Probabilty of drawing red balls $P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$
(ii) Probabilty of drawing 'not red ball' $P(\bar{A}) = 1 - P(A) = 1 - \frac{3}{8} = \frac{5}{8}$

9. **A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be (i) red ? (ii) white ? (iii) not green?**
Total number of marbles in abox = n(S) = 5 + 8 + 4 = 17
(i) Number of red marbles = n(A) = 5
Probability of taking out red marbles $P(A) = \frac{n(A)}{n(S)} = \frac{5}{17}$
(ii) Number of white marbles = n(B) = 8
Probabilty of taking out white marbles $P(B) = \frac{n(B)}{n(S)} = \frac{8}{17}$
(iii) Number of green marbles = n(C) = 4
Probabilty of taking out green marbles $P(C) = \frac{n(C)}{n(S)} = \frac{4}{17}$
∴ Probabilty of 'not green' marbles $P(C^1) = 1 - \frac{n(C)}{n(S)} = 1 - \frac{4}{17} = \frac{13}{17}$

10. **A piggy bank contains hundred 50p coins, fifty Rs1 coins, twenty Rs2 coins and ten Rs5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin (i) will be a 50 p coin ?(ii) will not be a Rs 5 coin?**

Total number of coins in a piggy bank = 100 + 50 + 20 + 10 = 180
Total number of 50 p coins = n(A) = 100

Number of Rs 5 coins = n(B) = 10

(i) Probabilty of getting Rs 5 coins $P(A) = \frac{n(A)}{n(S)} = \frac{100}{180} = \frac{5}{9}$

(ii)Probabilty of it will not be a Rs 5 coin $1 - P(B) = 1 - \frac{n(B)}{n(S)} = 1 - \frac{10}{180} = \frac{17}{18}$

11. Gopi buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fish and 8 female fish (see Fig. 15.4). What is the probability that the fish taken out is a male fish?


Fig. 15.4

Total number of fish in the tank = n(S) = 5+8 = 13

Number of male fish in the tank = n(A) = 5

The probability of taking out the male fish

$= P(A) = \frac{n(A)}{n(S)} = \frac{5}{13}$

12. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 (see Fig. 15.5), and these are equally likely outcomes. What is the probability that it will point at (i) 8 (ii) an odd number (iii) A number greater than 2 (iv) A number less than 9

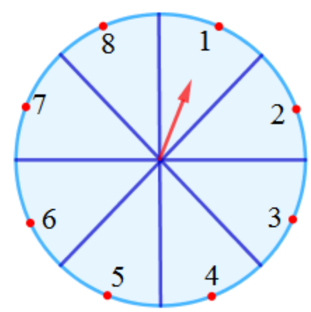


Fig 14.5

Possible number of events = 8

(i) Possible chances that an arrow pointing number 8 = 1

Probabilty of pointing 8 $= \frac{1}{8}$

(ii) Chances of pointing an odd number (1, 3, 5 ಮತ್ತು 7) = 4

Probabilty of pointing an odd number $= \frac{4}{8} = \frac{1}{2}$

(iii) Chances of pointing a number greater than 2
(i,e.3, 4, 5, 6, 7 and 8) = 6

Probablity of pointing a number greater than 2 $= \frac{6}{8} = \frac{4}{4}$

(iv) Chances of pointing less than 9 (i.e, 1,2,3,4,5,6,7,8) = 8

Probability of pointing a number less than 9 $= \frac{8}{8} = 1$

13. A die is thrown once. Find the probability of getting (i) a prime number; (ii) a number lying between 2 and 6; (iii) an odd number.

Possible numbers of events on throwing a dice = 6
Numbers on dice = 1,2,3,4,5 and 6
(i) Prime numbers = 2, 3 and 5
Favourable number of events = 3

Probability that it will be a prime number $= \frac{3}{6} = \frac{1}{2}$

(ii) Numbers lying between 2 and 6 = 3, 4 and 5
Favourable number of events = 3

Probability that a number between 2 and 6 $= \frac{3}{6} = \frac{1}{2}$

(iii) Odd numbers = 1, 3 and 5
Favourable number of events = 3

Probability that it will be an odd number $= \frac{3}{6} = \frac{1}{2}$

14. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting (i) a king of red colour (ii) a face card (iii) a red face card (iv) the jack of hearts (v) a spade (vi) the queen of diamonds

Possible numbers of events = 52

(i) Numbers of king of red colour = 2

Probability of getting a king of red colour = = $\frac{2}{52} = \frac{1}{26}$

(ii) Numbers of face cards = 12

Probability of getting a face card = $\frac{12}{52} = \frac{3}{13}$

(iii) Numbers of red face cards = 6

Probability of getting a king of red colour = $\frac{6}{52} = \frac{3}{26}$

(iv) Numbers of jack of hearts =1

Probability of getting a king of red colour = $\frac{1}{52}$

(v) Numbers of king of spade = 13

Probability of getting a king of red colour = $\frac{13}{52} = \frac{1}{4}$

(vi) Numbers of queen of diamonds = 1

Probability of getting a king of red colour = $\frac{1}{52}$

15. Five cards the ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random.
(i) What is the probability that the card is the queen?
(ii) If the queen is drawn and put aside, what is the probability that the second card picked up is (a) an ace? (b) a queen?

Total numbers of cards = 5

(i) Numbers of queen = 1

Probability of picking a queen = $\frac{1}{5}$

(ii) When queen is drawn and put aside then total numbers of cards left is 4

(a) Numbers of ace = 1

Probability of picking an ace = $\frac{1}{4}$

(a) Numbers of queen = 0

Probability of picking a queen = $\frac{0}{4} = 0$

16. 12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.

Numbers of defective pens = 12

Numbers of good pens = 132

Total numbers of pen = 132 + 12 = 144 pens

Favourable number of events = 132

Probability of getting a good pen $= \frac{132}{144} = \frac{11}{12}$

17. (i) A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective?
(ii) Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective?

(i) Total numbers of bulbs = 20
Numbers of defective bulbs = 4
Probability of getting a defective bulb = $\frac{4}{20} = \frac{1}{5}$
(ii) One non defective bulb is drawn in (i) then the total numbers of bulb left is 19
Total numbers of events = 19
Favourable numbers of events = 19 - 4 = 15
Probability that the bulb is not defective = $\frac{15}{19}$

18. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears (i) a two-digit number (ii) a perfect square number (iii) a number divisible by 5

Total numbers of discs = 50
(i) Total numbers of favourable events = 81
Probability that it bears a two-digit number = $\frac{81}{90} = \frac{9}{10}$
(ii) Perfect square numbers = 1, 4, 9, 16, 25, 36, 49, 64 and 81
Favourable numbers of events = 9; Probability of getting a perfect square number = $\frac{9}{90} = \frac{1}{10}$
(iii) Numbers which are divisible by 5 = 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85 and 90
Favourable numbers of events = 18; Probability of getting a number divisible by 5 = $\frac{18}{90} = \frac{1}{5}$

19. A child has a die whose six faces show the letters as given below:

A	B	C	D	E	A

The die is thrown once. What is the probability of getting (i) A? (ii) D?

Total numbers of events = 6
(i) Total numbers of faces having A on it = 2; Probability of getting A = $\frac{2}{6} = \frac{1}{3}$
(ii) Total numbers of faces having D on it = 1 ; Probability of getting A = $\frac{1}{6}$

20. Suppose you drop a die at random on the rectangular region shown in Fig. 15.6. What is the probability that it will land inside the circle with diameter 1m?[Not for examination]

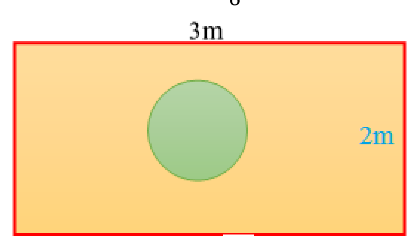


Fig 15.6

Area of the rectangle = $(3 \times 2)\ m^2 = 6m^2$
Area of the circle = $\pi r^2 = \pi\left(\frac{1}{2}\right)^2 = \frac{\pi}{4}\ m^2$
Probability that die will land inside the circle = $\frac{\frac{\pi}{4}}{6} = \frac{\pi}{24}$

21. A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that
(i) She will buy it? (ii) She will not buy it?

Total numbers of pens = 144
Numbers of defective pens = 20
Numbers of non defective pens = 144 - 20 = 124
(i) Numbers of favourable events = 124 ; Probability that she will buy it = $\frac{124}{144} = \frac{31}{36}$
(ii) Numbers of favourable events = 20; Probability that she will not buy it = $\frac{20}{144} = \frac{5}{36}$

22. Refer to Example 13. (i) Complete the following table

Event Sum on two dice	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$						$\frac{5}{36}$				$\frac{1}{36}$

(ii)A student argues that 'there are 11 possible outcomes 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12. Therefore, each of them has a probability $\frac{1}{11}$. Do you agree with this argument? Justify your answer.

Events that can happen on throwing two dices are (1,1), (1,2), (1,3), (1,4),(1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)

⇒ n(S) = 6x6 = 36

(i) To get sum as 2, possible outcomes = (1,1)

To get sum as 3, possible outcomes = (1,2) and (2,1)

To get sum as 4, possible outcomes = (1,3); (3,1); and (2,2)

To get sum as 5, possible outcomes = (1,4); (4,1); (2,3); and (3,2)

To get sum as 6, possible outcomes = (1,5); (5,1); (2,4); (4,2); and (3,3)

To get sum as 7, possible outcomes = (1,6); (6,1); (5,2); (2,5); (4,3); and (3,4)

To get sum as 8, possible outcomes = (2,6); (6,2); (3,5); (5,3); and (4,4)

To get sum as 9, possible outcomes = (3,6); (6,3); (4,5); and (5,4)

To get sum as 10, possible outcomes = (4,6); (6,4) and (5,5)

To get sum as 11, possible outcomes = (5,6) and (6,5)

To get sum as 12, possible outcomes = (6,6)

Event Sum on two dice	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

(ii) No, i don't agree with the argument. It is already justified in (i).

23. A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result i.e., three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game.

Events that can happen in tossing 3 coins
= HHH, HHT, HTH, THH, TTH, HTT, THT, TTT
Total number of events = 8
Hinif will lose the game if he gets HHT, HTH, THH, TTH, HTT, THT
Favourable number of elementary events = 6
Probability of losing the game = = $\frac{6}{8} = \frac{3}{4}$

24. A die is thrown twice. What is the probability that
(i) 5 will not come up either time? (ii) 5 will come up at least once?
[Hint : Throwing a die twice and throwing two dice simultaneously are treated as the same experiment]

(i) Total number of possibilities = 6x6 =36

Possible outcomes: (1,1), (1,2), (1,3), (1,4),(1,6), (2,1), (2,2), (2,3), (2,4), (2,6), (3,1), (3,2), (3,3), (3,4), (3,6), (4,1), (4,2), (4,3), (4,4),(4,6), (6,1), (6,2), (6,3), (6,4), (6,6)

The possibility of 5 will not come either time = 25

Required probability = = $\frac{25}{36}$

(ii) Number of events when 5 comes at least once = 11

Probability = $\frac{11}{36}$ [another way 1 - $\frac{25}{36} = \frac{11}{36}$]

25. Which of the following arguments are correct and which are not correct? Give reasons for your answer.
(i) If two coins are tossed simultaneously there are three possible outcomes—two heads, two tails or one of each. Therefore, for each of these outcomes, the probability is $\frac{1}{3}$
(ii) If a die is thrown, there are two possible outcomes—an odd number or an even number. Therefore, the probability of getting an odd number is $\frac{1}{2}$

(i) The statement is incorrect

Possible events = (H,H); (H,T); (T,H) and (T,T)

Probability of getting two heads = $\frac{1}{4}$

Probability of getting one of the each = $\frac{2}{4} = \frac{1}{2}$

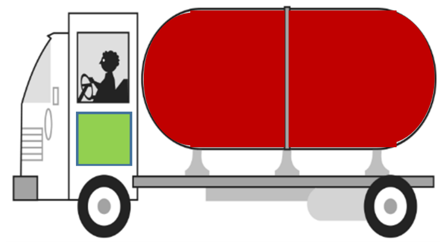
(ii) Correct. The two outcomes considered are equally likely.

Summary:

1. The difference between experimental probability and theoretical probability.
2. The theoretical (classical) probability of an event E, written as P(E), is defined as
   $$P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Number of all possible outcomes of the experiment}}$$
   where we assume that the outcomes of the experiment are equally likely.
3. The probability of a sure event (or certain event) is 1.
4. The probability of an impossible event is 0
5. The probability of an event E is a number P(E) such that
   $0 \le P(E) \le 1$
6. An event having only one outcome is called an elementary event. The sum of the probabilities of all the elementary events of an experiment is
7. For any event E, $P(E) + P(\overline{E}) = 1$ where E stands for 'not E'. E and E are called complementary events.

15 Surface Area and Volumes

You must have seen a truck with a container fitted on its back (see Fig. 15.2), carrying oil or water from one place to another. Is it in the shape of any of the four basic solids mentioned above? You may guess that it is made of a cylinder with two hemispheres as its ends.
A test tube, is also a combination of a cylinder and a hemisphere. Similarly, while travelling, you may have seen some big and beautiful buildings or monuments made up of a combination of solids mentioned above.



15.2 Surface Area of a Combination of Solids

To find the surface area or the volume of a container or test tube we have to break it up two or more known solids. For example,
Area of the container
= Area of the hemisphere + Area of the cylinder + Area of the hemisphere

Example 1 : Rasheed got a playing top (lattu) as his birthday present, which surprisingly had no colour on it. He wanted to colour it with his crayons. The top is shaped like a cone surmounted by a hemisphere (see Fig 13.6). The entire top is 5 cm in height and the diameter of the top is 3.5 cm. Find the area he has to colour. (Take $\pi = \frac{22}{7}$)

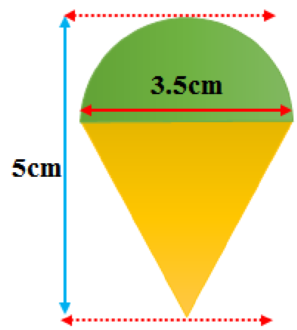


Fig 15.6

TSA of the toy = CSA of hemisphere + CSA of cone

CSA of hemisphere = $2\pi r^2 = \left(2\text{x } \frac{22}{7} \text{ x } \frac{3.5}{2} \text{ x } \frac{3.5}{2}\right) \text{cm}^2$

Height of the cone = Height of the top – Radius of the hemisphere

= 5 – 1.75 = 3.25cm

Slant height of cone (l) = $\sqrt{r^2 + h^2}$

= $\sqrt{(1.75)^2 + (3.25)^2} \approx 3.7$cm

∴ CSA of cone = π rl = $\left(\frac{22}{7} \text{ x } \frac{3.5}{2} \text{ x } 3.7\right) \text{cm}^2$

∴ TSA of the toy = $\left(2\text{x } \frac{22}{7} \text{ x } \frac{3.5}{2} \text{ x } \frac{3.5}{2}\right) + \left(\frac{22}{7} \text{ x } \frac{3.5}{2} \text{ x } 3.7\right) = \frac{22}{7} \text{ x } \frac{3.5}{2}(3.5 + 3.7)$

= 11 x 0.5(3.5 + 3.7) = 5.5 x 7.2 = 39.6cm^2

Example 2 : The decorative block shown in Fig. 13.7 is made of two solids — a cube and a hemisphere. The base of the block is a cube with edge 5 cm, and the hemisphere fixed on the top has a diameter of 4.2 cm. Find the total surface area of the block (Take $\pi = \frac{22}{7}$)

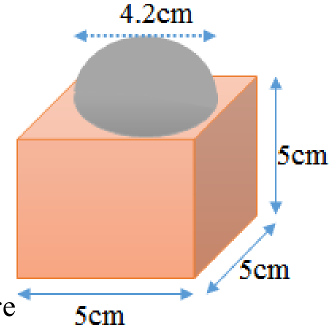


Fig 15.7

TSA of cube = 6a^2 = = 6 × (5 × 5) = 150 cm^2

The surface area of the block

= TSA of cube – base area of hemisphere + CSA of hemisphere

= (150 - πr^2 + 2 πr^2)cm^2 = (150 + πr^2)cm^2

= (150 + $\frac{22}{7}$x 2.1 x 2.1)cm^2 = (150 + 13.86)cm^2 = **163.86cm^2**

Example 3 : A wooden toy rocket is in the shape of a cone mounted on a cylinder, as shown in Fig. 15.8. The height of the entire rocket is 26 cm, while the height of the conical part is 6 cm. The base of the conical portion has a diameter of 5 cm, while the base diameter of the cylindrical portion is 3 cm. If the conical portion is to be painted orange and the cylindrical portion yellow, find the area of the rocket painted with each of these colours. (Take = 3.14)

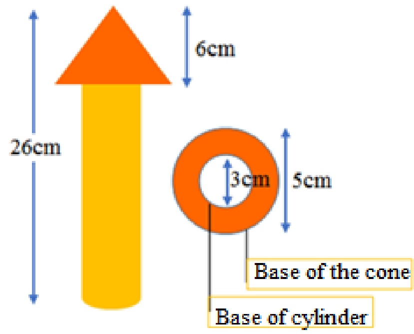


Fig 15.8

Let the the radius of the cone = r, the slant height = l the height of the cone = h , radius of the cylinder = r^1, and height of the cylinder = h^1

Then r = 2.5 cm, h = 6 cm, r^1 = 1.5 cm, h^1 = 26 - 6 = 20 cm ಮತ್ತು $l = \sqrt{r^2 + h^2}$

$= l = \sqrt{2.5^2 + 6^2} = 6.5$cm

Here, the conical portion has its circular base resting on the base of the cylinder, but the base of the cone is larger than the base of the cylinder. So, a part of the base of the cone (a ring) is to be painted.

The area to be painted orange

= CSA of the cone + base area of the cone– base area of the cylinder

$= \pi r l + \pi r^2 - \pi (r^1)^2$

$= \pi [(2.5 \times 6.5) + (2.5)^2 - (1.5)^2]$ cm^2

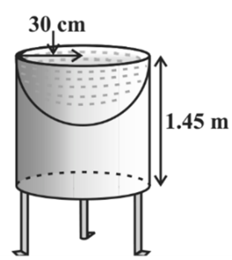
$= \pi [20.25]$ cm^2 = 3.14 × 20.25 cm^2 = **63.585 cm^2**

Now, the area to be painted yellow = CSA of the cylinder+ area of one base of the cylinder

$= 2\pi r^1 h^1 + \pi (r^1)^2$

$= \pi r^1 (2h^1 + r^1)$ = (3.14×1.5) (2×20 +1.5) cm^2 = 4.71× 41.5 cm^2 = **195.465 cm^2**

Example 4 : Mayank made a bird-bath for his garden in the shape of a cylinder with a hemispherical depression at one end (see Fig. 15.9). The height of the cylinder is 1.45 m and its radius is 30 cm. Find the total surface area of the bird bath (Take $\pi = \frac{22}{7}$).



Let h be height of the cylinder, and r the common radius of the cylinder and hemisphere. Then,

The total surface area of the bird-bath = CSA of cylinder + CSA of hemisphere

$= 2\pi rh + 2\pi r^2 = 2\pi r (h + r) = 2 \times \frac{22}{7} \times 30 (145 + 30)$ cm^2 = 33000 cm^2 = **3.3m^2**

Exercise 15.1

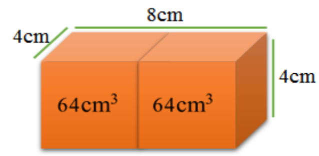
(Unless stated otherwise, take $\pi = \frac{22}{7}$)

1. **2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.**

The volume of the cube = 64cm^3, Therefore length of the side = 4cm

∴ The length of the cuboid = 4+4 = 8 cm

Breadth b = 4cm and height h = 4cm

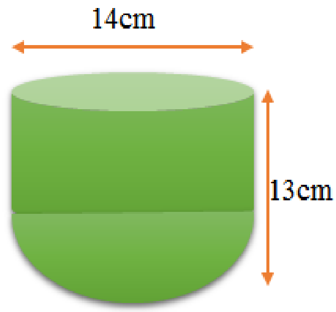


The surface area of the cuboid = 2(lb+bh+hl)

= 2(8x4+4x4+4x8)

= 2(32+16+32)= 2(80) = **160 cm²**

2. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.



The inner surface area of the vessel =

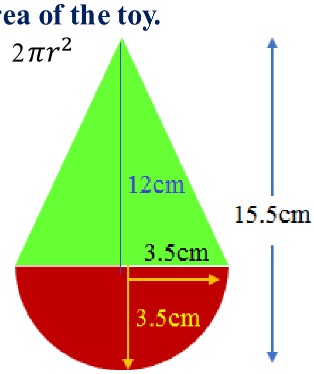
Inner surface area of the cylinder+ Inner surface area of the hemisphere = $2\pi rh + 2\pi r^2$

$\pi = \frac{22}{7}$; r = $\frac{14}{2}$ = 7cm; height of the cylinder h = 13 – 7 = 6cm

= 2x $\frac{22}{7}$ x 7x 6 + 2x$\frac{22}{7}$ x 7x 7 = 2x 22x 6 + 2x22 x 7

= 264 + 308 = 572cm²

3. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.



TSA of the Toy = CSA of cone + CSA of hemisphere = $\pi rl + 2\pi r^2$

$\pi = \frac{22}{7}$; r = 3.5 ; h = 15.5 – 3.5 = 12cm

$l = \sqrt{h^2 + r^2} = \sqrt{12^2 + 3.5^2} = \sqrt{144 + 12.25}$

$l = \sqrt{156.25}$ = 12.5cm

TSA of the Toy = $\frac{22}{7}$ x 3.5 x 12.5 + 2x $\frac{22}{7}$x3.5²

= 22 x 0.5 x 12.5 + 2x 22x0.5x3.5

= 11 x 12.5 + 11x7= 11 x 19.5

=214.5cm²

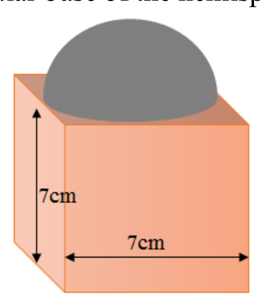
4. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid

The greatest diameter of the hemisphere = Side of the square = 7cm

Surface area of the solid

= Surface area of cube + CSA of hemisphere – The area of the circular base of the hemisphere

= $6a^2 + 2\pi r^2 - \pi r^2$



= $6x7^2 + 2 \text{ x } \frac{22}{7} \text{ x} \left(\frac{7}{2}\right)^2 - \frac{22}{7} \text{ x } \left(\frac{7}{2}\right)^2$

= 6x49 + 11x7 - 11 x $\frac{7}{2}$

= 294 + 77 - 11 x $\frac{7}{2}$

= 371 – 38.5

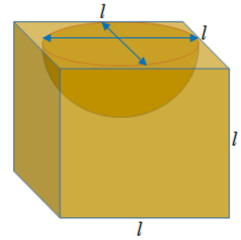
= 332.5cm²

5. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid

The surface area of the solid = Surface area of cube + Surface area of hemisphere

– Area of the circular base of the hemisphere

= $6l^2 + 2\pi r^2 - \pi r^2$



$= 6l^2 + 2\pi\left(\frac{l}{2}\right)^2 - \pi\left(\frac{l}{2}\right)^2$

$= 6l^2 + 2\pi\left(\frac{l}{2}\right)^2 - \pi\left(\frac{l}{2}\right)^2$

$= 6l^2 + \pi\left(\frac{l}{2}\right)^2 = \frac{l^2}{4}(24 + \pi)$

6. **A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see Fig. 13.10). The length of the entre capsule is 14mm and the diameter of the capsule is 5mm. Find its surface area.**

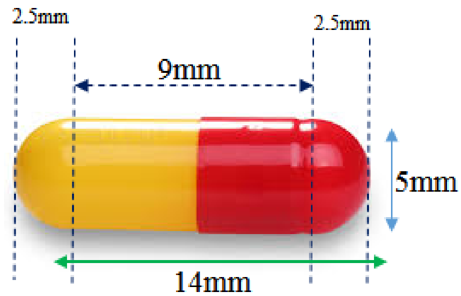


Fig 15.8

Surface area of the capsule =

2CSA of hemisphere CSA of cylinder = $2(2\pi r^2) + 2\pi rh$

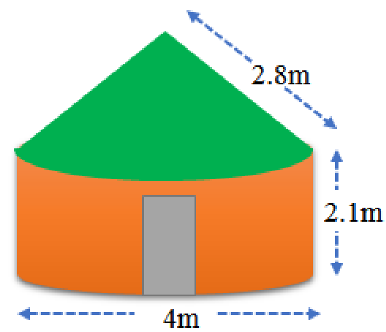
$\pi = \frac{22}{7}$; r = 2.5mm ; h = 9mm

$= 2(2\pi r^2) + 2\pi rh = 2\pi r(2r + h)$

$= 2 \text{x} \frac{22}{7} \text{ x } \frac{5}{2}(2\text{x}2.5 + 9)$

$= \frac{110}{7}(14) = 110 \text{ x } 2 =$ **220mm²**

7. **A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 500 per m². (Note that the base of the tent will not be covered with canvas.)**



The area of the tent

= CSA of the cylinder + CSA of cone

$= 2\pi rh + \pi rl$

$= \pi r(2h + l)$

$\pi = \frac{22}{7}$; r = 2m ; h = 2.1m ; l = 2.8m

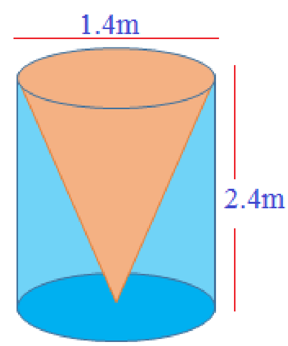
$= \frac{22}{7} \text{x } 2(2\text{x}2.1 + 2.8)$

$= \frac{44}{7} \text{x } 7\ (2\text{x}0.3 + 0.4)$

= 44 (0.6 + 0.4) = **44m²**

The total cost of the canvas at the rate of Rs 500/cm² = 44x500 = **Rs 22000**

8. **From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm²**



Surface area of the solid = TSA of cylinder + Inner CSA of cone

– Area of one the circular face of the cylinder

$= 2\pi r(r + h) + \pi rl - \pi r^2$

$\pi = \frac{22}{7}$; r = 0.7m ; h = 2.4m

$l = \sqrt{h^2 + r^2} = \sqrt{2.4^2 + 0.7^2} = \sqrt{5.76 + 0.49} = \sqrt{6.25} = 2.5$m

$= 2\text{x}\frac{22}{7}\text{x}0.7(0.7 + 2.4) + \frac{22}{7}\text{x } 0.7\text{x}2.5 - \frac{22}{7}\text{x}0.7\text{x}0.7$

$= 2\text{x}22\text{x}0.1(3.1) + 22\text{x } 0.1\text{x}2.5 - 22\text{x}0.1\text{x}0.7$

$= 4.4(3.1) + 2.2\text{x}2.5 - 2.2\text{x}0.7 = 13.64 + 5.5 - 1.54$

$= 13.64 + 5.5 - 1.54$ = **17.6m² ≈ 18m²**

9. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Fig. 15.11. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.

TSA of the article = CSA of cylinder + 2x innere CSA of hemisphere

$= 2\pi rh + 2x2\pi r^2$

$\pi = \frac{22}{7}$; r = 3.5cm ; h = 10m

$= 2\pi r(h + 2r)$

$= 2x\ \frac{22}{7}$ x 3.5 (10 + 2x3.5) = 2x 22 x 0.5 (10 + 7)

= 22 (17)= 22 (17) = **374cm²**

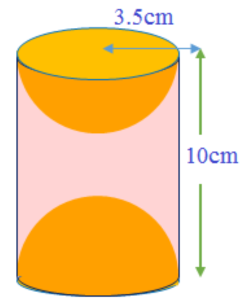


Fig 15.10

15.3 Volume of a Combination of Solids

Example 5 : Shanta runs an industry in a shed which is in the shape of a cuboid surmounted by a half cylinder (see Fig.15.12). If the base of the shed is of dimension 7 m × 15 m, and the height of the cuboidal portion is 8 m, find the volume of air that the shed can hold. Further, suppose the machinery in the shed occupies a total space of 300 m³, and there are 20 workers, each of whom occupy about 0.08 m³ space on an average. Then, how much air is in the sshed (Take $\pi = \frac{22}{7}$)

The volume of air inside the shed (when there are no people or machinery) is given by the volume of air inside the cuboid and inside the half cylinder, taken together.

Now, the length, breadth and height of the cuboid are 15 m, 7 m and 8 m, respectively. Also, the diameter of the half cylinder is 7 m and its height is 15 m

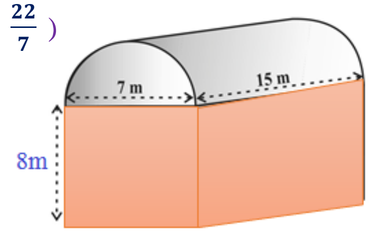


Fig 15.11

So, the required volume = volume of the cuboid + $\frac{1}{2}$ volume of the cylinder

$= \left[15 \times 7 \times 8 + \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 15\right] m^3$ = = **1128.75 m³**

Next, the total space occupied by the machinery = 300 m³

And the total space occupied by the workers = 20 × 0.08 m³ = 1.6m³

∴ the volume of the air, when there are machinery and workers

= 1128.86 - (300.00 + 1.60) = **827.15 m³**

Example 6:A juice seller was serving his customers using glasses as shown in Fig. 15.13. The inner diameter of the cylindrical glass was 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was 10cm, find the apparent capacity of the glass and its actual capacity. (Use π = 3.14)

Since the inner diameter of the glass = 5 cm and height = 10 cm,

the apparent capacity of the glass = $\pi r^2 h$

= 3.14 × 2.5 × 2.5 × 10 cm³ = 196.25 cm³

But the actual capacity of the glass is less by the volume of the hemisphere at the base of the glass.

i.e., it is less by $= \frac{2}{3}\pi r^3 = \frac{2}{3} \times 3.14 \times 2.5 \times 2.5 \times 2.5$ = **32.71 cm³**

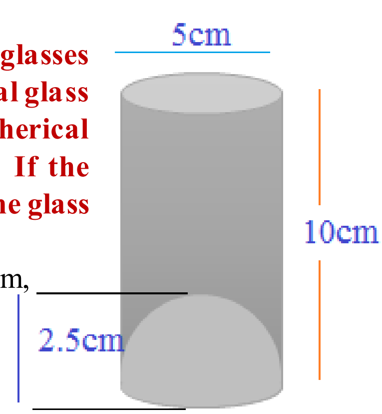


Fig 15.13

So, the actual capacity of the glass = apparent capacity of glass – volume of the hemisphere

= **(196.25 – 32.71) cm³**

Example 7: A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy. If a right circular cylinder circumscribes the toy, find the difference of the volumes of the cylinder and the toy. (take $\pi = 3.14$).

Height of the cylinder h = radius of the hemisphere + height of the cone = 2+2= 4cm

$\pi = 3.14$; radius of the hemisphere = radius of the cylinder = radius of the cone = 2cm

The volume of the cylinder circumsubscried the toy = $\pi r^2 h$

= 3.14 x 2 x 2 x 4 = 3.14 x 16 = 50.24cm^3

The volume of the toy = $\frac{2}{3}\pi r^3 + \frac{2}{3}\pi r^2 h$

= $\frac{1}{3}\pi r^2[2r + h]$ = $\frac{1}{3}$x 3.14x $2^2[4 + 2]$= $\frac{1}{3}$x 3.14x 4[6]

= $\frac{1}{3}$x 3.14x 4[6] = 25.12cm^3

Hence, the required difference of the two volumes

= 50.24 - 25.12 cm^3 = **25.12cm^3**

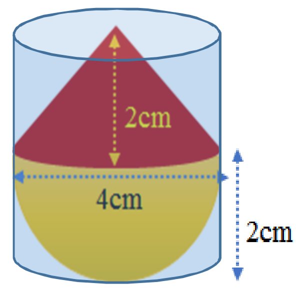


Fig 15.14

Exercise 15.2

[Unless stated otherwise, take $\pi = \frac{22}{7}$]

1. **A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π**

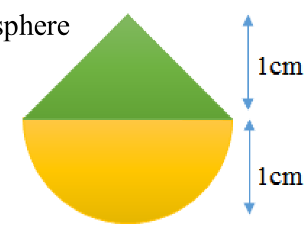
Volume of the solid = Volume of the cone + volume of the hemisphere

Given: $\pi = \frac{22}{7}$; h = 1cm; r = 1cm

Volume of the solid = $\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

= $\frac{1}{3}\pi$ x 1 x 1 + $\frac{2}{3}\pi$ x 1 x1 x1

=$\frac{2}{3}\pi + \frac{2}{3}\pi = \frac{3}{3}\pi = \pi$ cm^3



2. **Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly same]**

Volume of the air contained in the model = 2xVolume of the cone +Volume of the cylinder

V = $2\text{x}\frac{1}{3}\pi r^2 h_1 + \pi r^2 h_2$

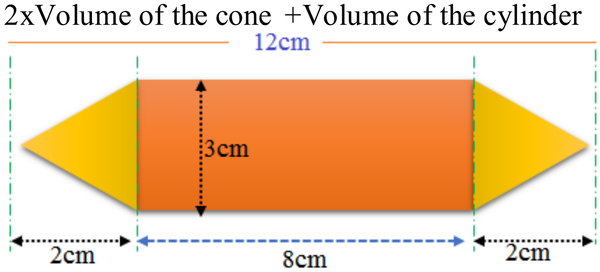
Given: $\pi = \frac{22}{7}$; r = 1.5cm;

h_1 = 2cm; h_2 = 8cm

V = $2\text{x}\frac{1}{3}\text{x}\frac{22}{7}\text{x}(1.5)^2\text{x}2 + \frac{22}{7}\text{x}(1.5)^2 8$

= $\frac{44}{21}$ x 2.25x2 + $\frac{22}{7}$x 2.25x8

= $\frac{44}{21}$ x 4.5 + $\frac{22}{7}$x 18 = $\frac{198}{21} + \frac{396}{7} = \frac{198}{21} + \frac{1188}{21} = \frac{1386}{21}$ = 66cm^3



3. **A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (see Fig. 15.15).**

Volume of the Jamun = 2 x Volume of two hemisphere + Volume of the cylinder

$= 2x\frac{2}{3}\pi r^3 + \pi r^2 h$

Given: $\pi = \frac{22}{7}$; r = 1.4cm; h = 2.2cm;

$= 2x\frac{2}{3}x\frac{22}{7}$ x 1.4 x 1.4 x 1.4 + $\frac{22}{7}$ x 1.4 x1.4 x 2.2

$= 4x\frac{22}{3}$x 0.2 x 1.4x1.4 + 22 x 0.2 x1.4 x 2.2

$= \frac{34.496}{3}$ + 13.552 = 11.5 + 13.552 = **25.05 cm³**

Therefore the amount of sugar contained = 25.05 x $\frac{30}{100}$ = **7.515cm³**

∴ The total amount of sugar contained in 45 jamun = 7.515x 45

= 338.175cm³ ≈ **338cm³**

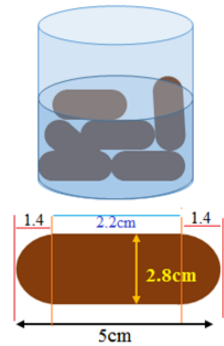


Fig 15.15

4. **A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (see Fig. 15.16)**

The radius of the conical deoressions r = 0.5cm, the depth h_1 = 1.4 cm

Length of the cuboid shape l = 15cm,breadth b = 10cm height ħ = 3.5cm

4 (Volume of the conical depressions) = $4\left(\frac{1}{3}\pi r^2 h_1\right)$

$= 4\left(\frac{1}{3}x\frac{22}{7}x\ 0.5\ x0.5\ x1.4\right) = 4\left(\frac{1}{3}x22x\ 0.5\ x0.5\ x\ 0.2\right)$

$= 4\left(\frac{1}{3}x22x\ 0.5\ x0.5\ x\ 0.2\right)$ = 1.47cm³

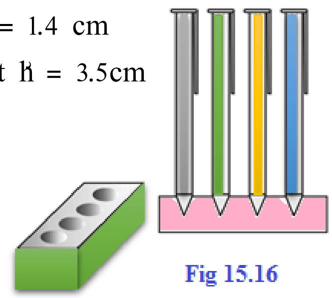


Fig 15.16

Volume of the wood in the pen stand

= Volume of the cuboid shape – 4 (Volume of the conical depressions)

= 15x10x3.5 – 1.47 = 525 – 1.47 = **523.53cm³**

5. **A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.**

The Volume of the lead shots = $\frac{4}{3}\pi r^3$

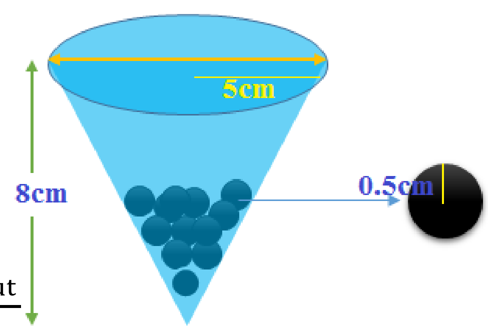
$= \frac{4}{3}\ x\ \frac{22}{7}\ x\ \frac{1}{2}\ x\ \frac{1}{2}\ x\ \frac{1}{2} = \frac{11}{21}$ cm³

The volume of the water in the vessel = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}x\frac{22}{7}x\ 5^2 x\ 8 = \frac{4400}{21}$ cm³

The volume of the water flows out

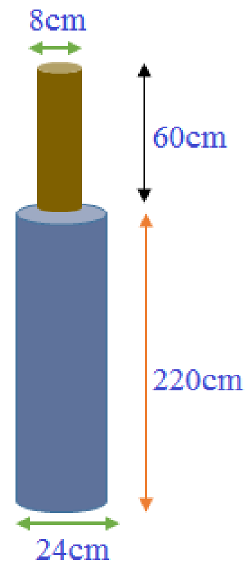
$= \frac{4400}{21}x\frac{1}{4} = \frac{1100}{21}$cm³



∴ Number of lead shots = $\frac{\text{Amount of water folws out}}{\text{Volume of the lead shot}}$

∴ Number of lead shots = $\frac{\frac{1100}{21}}{\frac{11}{21}}$ = 100 shots

6. **A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm³ of iron has approximately 8g mass. (Use = 3.14)**



r_1 = 8cm; $r_2 = \frac{24}{2}$ = 12cm; h_1 = 60cm; h_2 = 220cm

Volume of the pole

= Volume of the first cylinder + Volume of the second cylinder

$= \pi r_2^2 h_2 + \pi r_1^2 h_1$

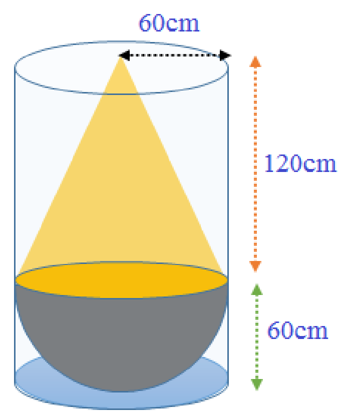
= 3.14 x 12 x 12 x 220 + 3.14 x 8 x 8 x 60

= 99475.2 + 12057.6 = 111532.8cm^3

The mass of the iron / 1 cm^3 = 8g

∴ Mass of the iron pole = 111532.8 x 8 = 892262.4 g = 892.26kg

7. **A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.**



Radius of the cylinder r = 60cm; height h = 180cm

Height of the cone $h_1 = 120cm$

Volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7}$x 60 x 60 x 180 = 2036571.43cm^3

Volume of the cone $= \frac{1}{3}\pi r^2 h_1$

$= \frac{22}{7}$ x 20 x 60 x 120 = 452571. 43cm^3

Volume of the hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3}$ x $\frac{22}{7}$ x 60 x60 x60 = 4525571.43cm^3

∴ The volume of the water left in the cylinder

= 2036571.43 – (452571.43 + 452571.43) = 2036571.43 – 905142.86

= 1131428.57cm^3 = **1.131m^3**

Alternate method:

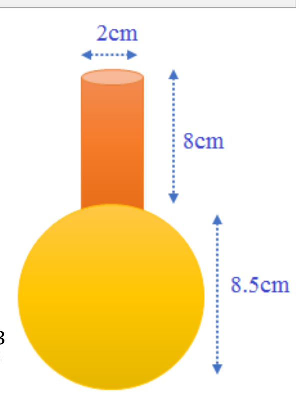
The volume of the water left in the cylinder

$= \left[\pi r^2 h - \frac{1}{3}\pi r^2 h_1 + \frac{2}{3}\pi r^3\right] = \pi r^2\left[h - \frac{1}{3}h_1 + \frac{2}{3}r\right]$

$= \frac{22}{7}$x 60x60 $\left[180 - \frac{1}{3}\text{x}120 + \frac{2}{3}\text{x}60\right] = \frac{22}{7}$x 60x60[180 – (40 + 40)]

$= \frac{22}{7}$x 60x60x 100 = = 1131428.57cm^3 = 1.131m^3

8. **A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements, and π = 3.14.**



Height of the cylinder h = 8cm; Radius $r_1 = \frac{2}{2}$ = 1cm

Radius of the sphere $r_2 = \frac{8.5}{2}$ cm

Volume of the Vessel

= Volume of the cylinder + Volume of the sphere $= \pi {r_1}^2 h + \frac{4}{3}\pi r_2^3$

$= 3.14\text{x}1^2\text{x } 8 + \frac{4}{3}\text{x}3.14\text{x}\left(\frac{8.5}{2}\right)^3 = 25.12 + \frac{11}{21}$x 8.5x8.5x8.5

= 25.12 + 321.39 = **346.51cm^3**

So, there is little difference in her measurment

13.4 Conversion of Solid from One Shape to Another

We can convert one shape to another. When we convert the shape, the volume of the new shape will be the same as the earliar shape.

Example 8: A cone of height 24 cm and radius of base 6 cm is made up of modelling clay. A child reshapes it in the form of a sphere. Find the radius of the sphere.

Volume of the cone = $\frac{1}{3} \times \pi \times 6 \times 6 \times 24$ cm^3

If the radius of the sphere is `r ' then the volume of the sphere is $\frac{4}{3}\pi r^3$

Therefore, Volume of the cone = Volume of the sphere

$\Rightarrow \frac{4}{3}\pi r^3 = \frac{1}{3} \times \pi \times 6 \times 6 \times 24$ cm^3

$r^3 = 3 \times 3 \times 24 = 3^3 \times 2^3 \Rightarrow r = 3 \text{ x } 2 = 6$

Radius of the sphere = 6 cm.

Example 9: Selvi's house has an overhead tank in the shape of a cylinder. This is filled by pumping water from a sump (an underground tank) which is in the shape of a cuboid. The sump has dimensions 1.57 m × 1.44 m × 95cm. The overhead tank has its radius 60 cm and height 95 cm. Find the height of the water left in the sump after the overhead tank has been completely filled with water from the sump which had been full. Compare the capacity of the tank with that of the sump (use π = 3.14)

Volume of water in the overhead tank = Volume of the water removed from the sump.

Now, the volume of water in the overhead tank (cylinder) = $\pi r^2 h$

= $3.14 \times 0.6 \times 0.6 \times 0.95$ m^3

The volume of water in the sump when full = $l \times b \times h = 1.57 \times 1.44 \times 0.95$ m^3

The volume of water left in the sump after filling the tank

= $[(1.57 \times 1.44 \times 0.95) - (3.14 \times 0.6 \times 0.6 \times 0.95)]$ m^3 = $(1.57 \times 0.6 \times 0.6 \times 0.95 \times 2)$ m^3

So, the height of the water left in the sump h = $\frac{\text{volume of water left in the sump}}{l\text{xb}}$

= $\frac{1.57 \times 0.6 \times 0.6 \times 0.95 \times 2}{1.57\text{x}1.44}$ m = 0.475 m = 47.5 cm

Also , $\frac{\text{Capacity of the tank}}{\text{capacity of the sump}} = \frac{3.14 \text{ x } 0.6 \text{ x } 0.6 \text{ x } 0.95}{1.57 \text{ x } 1.44 \text{ x } 0.95} = \frac{1}{2}$

Therefore, the capacity of the tank is half the capacity of the sump

Example 10 : A copper rod of diameter 1 cm and length 8 cm is drawn into a wire of length 18 m of uniform thickness. Find the thickness of the wire.

The volume of the rod = $\pi \times \left(\frac{1}{2}\right)^2 \text{x } 8$ cm^3 = $2\,\pi$ cm^3

The length of the new wire of the same volume = 18 m = 1800 cm

If r is the radius (in cm) of cross-section of the wire, its volume = $\pi r^2 \times 1800$ cm^3

Therefore, $\pi r^2 \times 1800 = 2\pi \Rightarrow r^2 = \frac{1}{900} \Rightarrow r = \frac{1}{30}$

So, the diameter of the cross section, i.e., the thickness of the wire is $\frac{1}{15}$cm

i.e., Approximately 0.67 mm

Example 11 : A hemispherical tank full of water is emptied by a pipe at the rate of $3\frac{4}{7}$ litres per second. How much time will it take to empty half the tank, if it is 3m in diameter? (Take = $\frac{22}{7}$)

Radius of the hemispherical tank = $\frac{3}{2}$ m

Volume of the tank $= \frac{2}{3} \times \frac{22}{7} \times \frac{3}{2}\ m^3 = \frac{99}{14}\ m^3$

So, the volume of the water to be emtied $= \frac{1}{2} \times \frac{99}{14}\ m^3 = \frac{99}{28} \times 1000 = \frac{99000}{28}\ l$tr

since, $\frac{25}{7}$ ltr of water emptied in 1 second

The required time to empty $\frac{99000}{28}$ ltrs of water $= \frac{99000}{28} \times \frac{7}{25}$ seconds = 16.5minute

Exercise 15.3

(Take $\pi = \frac{22}{7}$ unless stated otherwise)

1. **A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.**

Radius of the sphere r_1 = 4.2cm, Radius of the cylinder r_2 = 6cm

$\frac{4}{3}\pi {r_1}^3 = \pi r_2^2 h$

$= \frac{4}{3} \times 4.2^3 = 6^2 h \Rightarrow 4 \times 1.4 \times 4.2 \times 4.2 = 36h \Rightarrow 4 \times 1.4 \times 4.2 \times 4.2 = 36h$

$\Rightarrow 98.784 = 36h \Rightarrow h = 2.744$cm

2. **Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.**

$r_1 = 6$cm, $r_2 = 8$cm, $r_3 = 10$cm

Let radius of the resulting sphere = r

$\frac{4}{3}\pi r^3 = \left(\frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3\right) \Rightarrow \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(r_1^3 + r_2^3 + r_3^3)$

$\Rightarrow r^3 = (6^3 + 8^3 + 10^3) \Rightarrow r^3 = (216 + 512 + 1000)$

$\Rightarrow r^3 = 1728 \Rightarrow r = 12$cm

3. **A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.**

Depth of the well h = 20m; radius of the well $r = \frac{7}{2}$ m

Length of the platform l = 22m; Breadth b = 14m; Height H = ?

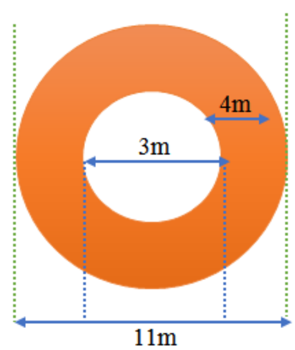
Volume of the well = Volume of the platform

$\pi r^2 h = lbh$

$\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 20 = 22\ m \times 14h$

$\Rightarrow 11 \times 7 \times 10 = 22\ m \times 14h \Rightarrow 10 = 4h \Rightarrow 14h = 22 \times 7 \times 5 \Rightarrow h = \frac{10}{4} = 2.5$m

4. **A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment .**



Volume of the well $= \pi r^2 h = \frac{22}{7} \times 1.5 \times 1.5 \times 14$

$= 22 \times 2.25 \times 2 = 44 \times 2.25 = 99cm^3$

Volume of the embankment $= \frac{22}{7}\left[\frac{11}{2} \times \frac{11}{2} - \frac{3}{2} \times \frac{3}{2}\right]h$

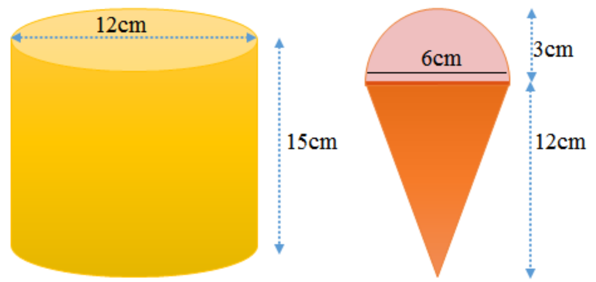
$= \frac{22}{7}h\ [30.25 - 2.25] = \frac{22}{7}\ [28]h = 88h$

Volume of the embankment = Volume of the well

$88h = 99 \Rightarrow h = \frac{99}{88} = 1.125 \Rightarrow \frac{2662}{7}h = 99$

$\Rightarrow h = \frac{99 \times 7}{2662} = 0.26$m

5. A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream



Volume of ice cream to be filled in cone =

Volume of Hemisphere + Volume of cone

$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$= \frac{2}{3} \text{x } \frac{22}{7} \text{x } 3^3 + \frac{1}{3} \text{x } \frac{22}{7} \text{ x } 3 \text{ x } 3 \text{ x } 12$

$= 2\text{x } \frac{22}{7} \text{x } 9 + \frac{22}{7} \text{ x } 3 \text{ x } 12$

$= \frac{396}{7} + \frac{792}{7} = \frac{1188}{7} \text{cm}^3$

The ice-cream filled in the cylinder = Volume of the cylinder $= \frac{22}{7} \text{x } 6 \text{ x } 6 \text{ x } 15 = \frac{11880}{7} \text{ cm}^3$

Therefore, number of cones $= \frac{\frac{11880}{7}}{\frac{1188}{7}} = \frac{11880}{1188} = 10$

6. How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions 5.5 cm × 10 cm × 3.5 cm?

No.of silver coins $= \frac{\text{Volume the cuboid}}{\text{Volume of silver coin}} = \frac{\text{lbh}}{\pi r^2 h}$

Volume of silver coin lbh = 5.5 x 10 x 3.5 = 192.5cm^3

Volume of cuboid $\pi r^2 h = \frac{22}{7} x \frac{1.75}{2} x \frac{1.75}{2} x\ 0.2 = 0.48125$

No.of silver coins $= \frac{192.5}{0.48125} = 400$

7. A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

Height of the cylindrical bucket h = 32cm; radius of the base = 18cm

Height of heap of the sand H = 24cm

Volume of cylindrical bucket $= \pi r^2 h = \frac{22}{7} \text{ x } 18 \text{ x}18 \text{ x } 32 = \frac{228096}{7}$

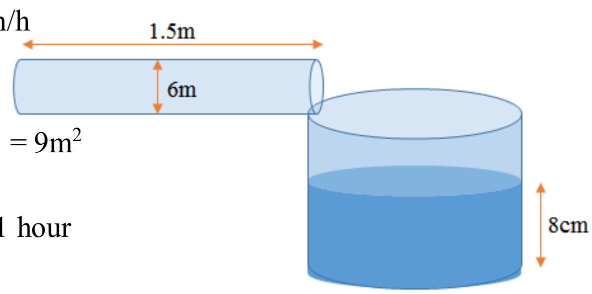
Volume of heap of sand $\frac{1}{3}\pi r^2 \text{H} = \frac{1}{3} \text{ x } \frac{22}{7} \text{ x } r^2 \text{ x } 24 = \frac{528}{21} r^2$

Volume of heap of sand = Volume of cylindrical bucket

$\frac{528}{21} r^2 = \frac{228096}{7} \Rightarrow r^2 = \frac{228096\text{x}21}{7\text{x}528} = 1296$

$l = \sqrt{24^2 + 36^2} = \sqrt{576 + 1296} = \sqrt{1872} = \sqrt{144\text{x}13} = 12\sqrt{13}$ cm

8. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?



Speed of the water = 10km/h ⇒ 10 x 1000 m/h

Length of the water flows in 1 hour

l = 10 x 1000 m

Area of the rectangular canal = lb = 6 x 1.5 = 9m^2

The volume of water flows in 1 hour

= Area of canal x Length of water flows in 1 hour

= 9 x 10 x1000 m^3

The volume of water flows in 30 minutes = $\frac{9 \times 10 \times 1000 \text{ m3}}{2}$ = 45000m^3

Hence, the area required for covering 8 cm = $\frac{8}{100}$ m of standing water

= $\frac{450000}{8}$ x 100 = 562500m^3 = 56.25hec [1hec = 10000m^3]

9. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

Speed of the water = 10km/h ⇒ 3 x 1000 m/h

Length of the water flows in 1 hour l = 3000 m

Now, area of the pipe which is in the form of a circle = πr^2

= π x $\left(\frac{20}{2}\right)^2$ = 100 π cm^2 = $\frac{1}{100}\pi$ m^2

Volume of cylindrical tank = $\pi r^2 h$ = π x 5 x 5 x 2 = 50 π m^2

∴ Required time = $\frac{\text{Volume of cylindrical tank}}{\text{area of the pipe which is in the form of a circle x Length of the water}}$

= $\frac{50\pi}{\frac{1}{100}\pi \times 3000}$ = $\frac{50 \times 100}{3000}$ = 1.67 hours or 1.67x 60 = 100 min

15.5 Frustum of a Cone

Given a cone, when we slice (or cut) through it with a plane parallel to its base (see Fig. 15.20) and remove the cone that is formed on one side of that plane, the part that is now left over on the other side of the plane is called a frustum of the cone.

Example 12 : The radii of the ends of a frustum of a cone 45 cm high are 28 cm and 7 cm (see Fig. 15.21). Find its volume, the curved surface area and the total s uface area (take $\pi = \frac{22}{7}$)

Volume of frustum of cone = $\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

CSA of frustum of cone = $\pi(r_1 + r_2)l$ [$l = \sqrt{h^2 + (r_1 - r_2)^2}$

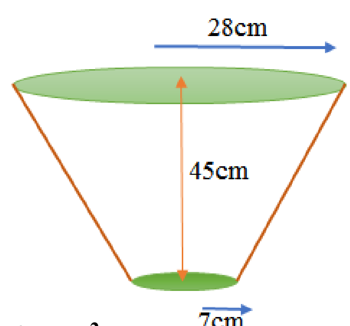
TSA of frustum of cone = $\pi(r_1 + r_2)l + \pi r_1^2 + \pi r_2^2$

h = 48cm, r_1 = 28cm, r_2 = 7cm

$l = \sqrt{h^2 + (r_1 - r_2)^2}$

$\Rightarrow l = \sqrt{48^2 + (28-7)^2}$ $\Rightarrow l = \sqrt{(3\text{x}15)^2 + (3\text{x}7)^2}$

$\Rightarrow l = 3\sqrt{225 + 49}$ $\Rightarrow l = 3\sqrt{225 + 49}$ = **49.65cm**



i) CSA of frustum of cone = $\pi(r_1 + r_2)l$

= $\frac{22}{7}(28 + 7)49.65$ = 22 x 5 x 49.65 = **5461.5cm^2**

ii) TSA of frustum of cone = $\pi(r_1 + r_2)l + \pi r_1^2 + \pi r_2^2$

= $\frac{22}{7}(28 + 7)49.65$ + $\frac{22}{7}$ x 28 x 28+ $\frac{22}{7}$ x 7 x 7

= 5461.5 + 22 x 4 x 28+ 22 x 7 = 5461.5 + 2464+ 154= **8079.5cm^2**

iii) Volume of frustum of cone = $\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

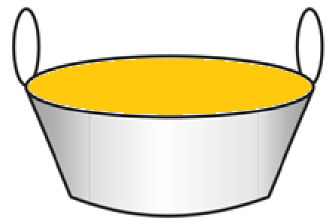
= $\frac{1}{3}$ x $\frac{22}{7}$x 45 (28x28 + 7x7 + 28x7) = $\frac{22}{7}$x 15(784 + 49 + 196)

= $\frac{22}{7}$x 15(784 + 49 + 196) = **48510cm^3**

Example 13 : Hanumappa and his wife Gangamma are busy making jaggery out of sugarcane juice. They have processed the sugarcane juice to make the molasses, which is poured into moulds in the shape of a frustum of a cone having the diameters of its two circular faces as 30 cm and 35 cm and the vertical height of the mould is 14 cm (see Fig. 15.22). If each cm^3 of molasses has mass about 1.2 g, find the mass of the molasses that can be poured into each mould(take $\pi = \frac{22}{7}$)

Since the mould is in the shape of a frustum of a cone, the quantity (volume) of molasses that can be poured into it = $\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$ where, h = 14cm, $r_1 = \frac{35}{2}$, $r_2 = 15$

Volume = $\frac{1}{3}$ x $\frac{22}{7}$ x 14(17.5x17.5 + 15x15 + 17.5x15)



= $\frac{1}{3}$ x 22 x 2(17.5x17.5 + 15x15 + 17.5x15)

= $\frac{1}{3}$ x 22 x 2(306.25 + 225 + 262.5)

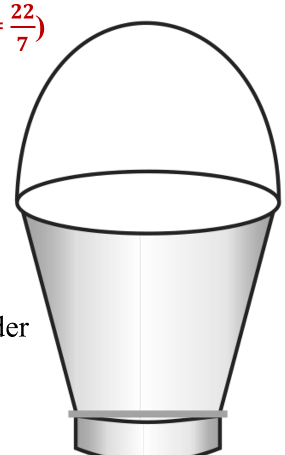
= $\frac{1}{3}$ x 22 x 2(793.75) = 11,641.7cm^3

Mass of molasses = 1.2 g

∴ The mass of the molasses that can be poured into each mould = (11641.7 × 1.2)g

= 13970.04 g = 13.97 kg ≈ **14 kg**

Example 14 : An open metal bucket is in the shape of a frustum of a cone, mounted on a hollow cylindrical base made of the same me- tallic sheet (see Fig. 15.23). The diameters of the two circular ends of the bucket are 45 cm and 25 cm, the total vertical height of the bucket is 40 cm and that of the cylindrical base is 6 cm. Find the area of the metallic sheet used to make the bucket, where we do not take into account the handle of the bucket. Also, find the volume of water the bucket can hold.(Take $\pi = \frac{22}{7}$)



Height of the bucket = 40 cm,(it include height of the base)

Height of the frustum of cone = h = (40 - 6) cm = 34 cm

Slant height of frustum of cone $l = \sqrt{h^2 + (r_1 - r_2)^2}$

Where, h = 34cm, $r_1 = 22.5cm$, r_2 = 12.5cm

$l = \sqrt{34^2 + (22.5 - 12.5)^2} \Rightarrow l = \sqrt{34^2 + (10)^2} = 35.44cm$

The area of metallic sheet used

= CSA of frustum of cone + Area of circular base + CSA of cylinder

= [π × 35.44 (22.5 + 12.5) + π × (12.5)2 + 2 π × 12.5 × 6] cm^2

= $\frac{22}{7}$(1240.4 + 156.25 + 150) cm^2 = **4860.9 cm^2**

Now, the volume of water that the bucket can hold (also, known as the capacity of the bucket)

= $\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

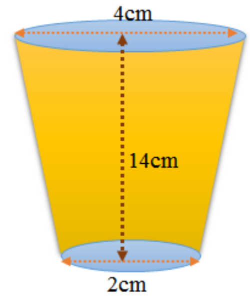
= $\frac{1}{3}$x $\frac{22}{7}$ x 34 (22.5^2 + 12.5^2 + 22.5x12.5) = $\frac{748}{31}$ (506.25 + 156.25 + 281.25)

= $\frac{748}{31}$ (943.75) = 33615.48cm^3 = 33.62 ಲೀಟರ್‌ಗಳು (ಸರಿಸುಮಾರಾಗಿ)

Exercise 15.4

(use $\pi = \frac{22}{7}$ unless stated otherwise)

1. **Now, the volume of water that the bucket can hold (also, known as the capacity of the bucket)**



Volume of the bucket that hold water

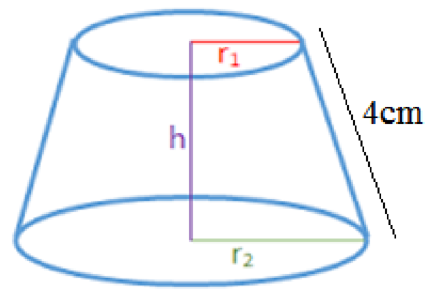
$= \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

$\pi = \frac{22}{7}$; h = 14cm; $r_1 = \frac{4}{2}$ = 2cm; r_2 $\frac{2}{2}$ = 1cm

$= \frac{1}{3} x \frac{22}{7} x 14(4 + 1 + 2)$

$= \frac{1}{3} x \frac{22}{7} x 14(7) = \frac{1}{3} x 22 x 14 =$ **$102\frac{2}{3}$ cm²**

2. **The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum**



Circumference of the circular base = 18cm

$\Rightarrow 2\pi r_1 = 18 \Rightarrow r_1 = \frac{9}{\pi}$ cm

Circumference of the circular top = 6cm

$\Rightarrow 2\pi r_2 = 6 \Rightarrow r_2 = \frac{3}{\pi}$ cm

CSA of frustum of cone $= \pi(r_1 + r_2)l$

$= \pi(r_1 + r_2)l$

$= \pi\left(\frac{9}{\pi} + \frac{3}{\pi}\right)4 = \pi\left(\frac{9+3}{\pi}\right)4 = 48\text{cm}^3$

3. **A fez, the cap used by the Turks, is shaped like the frustum of a cone (see Fig. 15.24). If its radius on the open side is 10 cm, radius at the upper base is 4 cm and its slant height is 15 cm, find the area of material used for making it.**

TSA of fez = CSA of fez + Area of circular top

CSA of frustum of cone = $\pi(r_1 + r_2)l + \pi r_2^2$

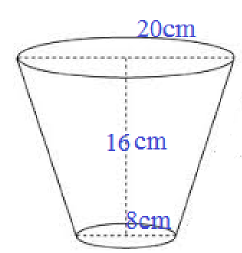
r_1 = 10cm; r_2 = 4cm ; l = 15cm

$= \frac{22}{7}(10 + 4)15 + \frac{22}{7} x 4^2$

$= \frac{22}{7}(14)15 + \frac{22}{7} x 16 = \frac{4620}{7} + \frac{352}{7}$

$= \frac{4972}{7} = 710\frac{2}{7}\text{cm}^3$

4. **A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of Rs 20 per litre. Also find the cost of metal sheet used to make the container, if it costs Rs 8 per 100 cm². (Take =3.14)**



Volume of frustum of cone $= \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

$= \frac{1}{3} x 3.14 x 16(8 x 8 + 20 x 20 + 8x20)$

$= \frac{1}{3} x 3.14 x 16(64 + 400 + 160)$

$= \frac{1}{3} x 3.14 x 16(624) = 10449.9\text{cm}^2 \Rightarrow 10.45$ ltr

Total amount required at the rate of Rs 20/ltr = 10.45 x 20 = Rs 209

$l = \sqrt{h^2 + (r_1 - r_2)^2}$

$l = \sqrt{16^2 + (8-20)^2} = l = \sqrt{256 + 144} = \sqrt{400} = 20$cm

TSA of frustum of cone

= CSA of frustum of cone + Area of the circular bottom = $\pi(r_1 + r_2)l + \pi r_1^2$

= $3.14(8 + 20)20 + 3.14 \times 8^2 = 3.14(28)20 + 3.14 \times 64$

= $3.14(28)20 + 3.14 \times 64 = 1758.4 + 200.96 = 1{,}959.36\text{cm}^2$

Cost of metal for 100cm^2 = Rs 8

Therefore total cost of metals used = $\frac{1{,}959.36}{100} \times 8$ = **Rs 156.75**

5. A metallic right circular cone 20 cm high and whose vertical angle is 60° is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter $\frac{1}{16}$cm find the length of the wire.

Cot $30^0 = \frac{AO}{BO} \Rightarrow \sqrt{3} = \frac{10}{BO} \Rightarrow BO = \frac{10}{\sqrt{3}}$ cm = r_1

Cot $30^0 = \frac{AD}{CD} \Rightarrow \sqrt{3} = \frac{20}{CD} \Rightarrow CD = \frac{20}{\sqrt{3}}$ cm = r_2

Volume of the frustum of cone = $\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

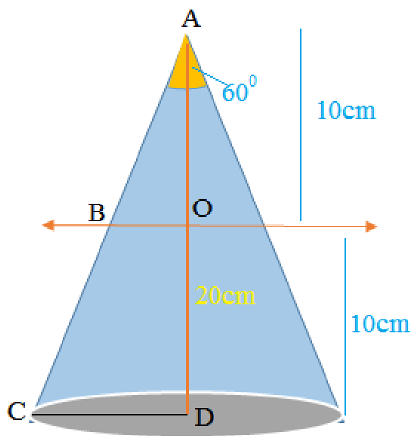
= $\frac{1}{3} \times \frac{22}{7} \times 10 \left[\left(\frac{10}{\sqrt{3}}\right)^2 + \left(\frac{20}{\sqrt{3}}\right)^2 + \frac{10}{\sqrt{3}} \times \frac{20}{\sqrt{3}}\right]$

= $\frac{\pi}{3} \times 10 \left[\frac{100}{3} + \frac{400}{3} + \frac{200}{3}\right] = \frac{\pi}{3} \times 10 \left[\frac{700}{3}\right] = \mathbf{\frac{7000\pi}{9}}$

Volume of the wire = Volume of the frustum of cone

$\pi r^2 h = \frac{7000\pi}{9} \Rightarrow \pi \left(\frac{1}{32}\right)^2 h = \frac{7000\pi}{9}$

$\Rightarrow \frac{1}{1024}h = \frac{7000}{9} \Rightarrow h = \frac{7000 \times 1024}{9} \Rightarrow h = 796444.44\text{cm} = 7964.44\text{m}$



Summary:

1. To determine the surface area of an object formed by combining any two of the basic solids, namely, cuboid, cone, cylinder, sphere and hemisphere.
2. To find the volume of objects formed by combining any two of a cuboid, cone, cylinder, sphere and hemisphere.
3. Given a right circular cone, which is sliced through by a plane parallel to its base, when the smaller conical portion is removed, the resulting solid is called a Frustum of a Right Circular Cone.
4. The formulae involving the frustum of a cone are

   Volume of frustum of cone = $\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

   CSA of frustum of cone = $\pi(r_1 + r_2)l$ [$l = \sqrt{h^2 + (r_1 - r_2)^2}$

   TSA of frustum of cone = $\pi(r_1 + r_2)l + \pi r_1^2 + \pi r_2^2$